STABILIZATION OF LINEAR CONTROL SYSTEMS OVER MARKOV COMMUNICATION CHANNELS

BY

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بسم الله الرحمن الرحيم

This Thesis is dedicated

to

My parents

for their duas, constant support and encouragement throughout my life.

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In the name of Allah, the Most Beneficent, the Most Merciful.

All praise to Allah, the Cherisher and Sustainer of the worlds, none is worthy of worship but Him. Blessings and peace upon the Prophet (SAWS), the mercy for all creatures. My thanks to King Fahd University of Petroleum and Minerals for providing a great environment for education and research. I would also like to thank the Deanship for Scientific Research (DSR) at KFUPM for financial support through research group project RG-1105-1.

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THESIS ABSTRACT

Name: Gulam Dastagir Khan.

Title: Stabilization of Linear Systems Over Markov communication channels

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This thesis is mainly concerned with the analysis and design of networked control systems (NCSs) with packet dropouts and delays in communication channels which vary in a random fashion. In the past, several approaches have been used to model network delays. In this thesis, networks control systems have been analysed by modeling the delay using Markovian chains, which is a relatively fresh and complicated approach to this problem. A wide range of recent research activities related to the problem of distributed control systems have been reported in this thesis. The introduction of delays and dropouts in the communication channels is inevitable. Various reasons for these dropouts and delays are limited bandwidth, losses and also due to overhead in communication nodes and in the network. These network anomalies threaten the stability and over all performance of a physical system. This research mainly deals with the investigation of system stability, controller synthesis and derivation for the sufficient condition for which the closed loop NCS is stable. Markov jump linear systems with packets dropouts have been studied in detail for both

completely known and partially known transition probability matrices, following which, some typical examples have been provided to show the effectiveness of developed methodologies.

مخلص الرسالة

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تتصدى هذه الرساله لتحليل و تصميم نظم تحكم الشبكات فى ضوء حدوث تاخير و تساقط حزم بيانات فى قنوات اتصال بصورة عشوائيه. وتتوافرعدة طرائق فى المقالات العلميه المنشورة لنمذجه التأخير فى شبكات التحكم. وعليه فتتناول هذه الرساله تحليل و تصميم نظم تحكم الشبكات مع نمذجه التأخير بطريقه سلاسل ماركوف بهدف اضفاء الشمولية والمرونه على الاداء . و يعتبر هذا الاسلوب حديثا و مثمرا ويعالج مشاكل عديده مثل محدودية نطاق الشبكات و فقد البيانات فى قنوات الاتصال مما يؤثر على اتزان نظم تحكم الشبكات أو تدهور الاداء بصوره كبيرة.

وفى هذا الصدد تطور الرساله اسلوبا جديدا لتحليل نظم تحكم الشبكات عبر قنوات اتصال ماركوف باستخدام الرياضيات المتقدمه و تستنبط شروط تصميم فعاله لضمان اتزانيه نظم العروة المغلقه. كما تدرس تحليل نظم تحكم الشبكات عبر قنوات اتصال ماركوف فى ضوء تغير المعاملات بصوره عشوائيه جزئيه أو كليه. وقد تم تطبيق الاساليب المستنبطه على نظم عمليه و اثبتت النتائج دقه الاساليب و تفوقها على مثيلاتها.

Chapter 1

INTRODUCTION

1.1 Introduction To Networked Control Systems

The control theory is largely based on the idea that information (signals) are transmitted along perfect communication channels and that computation is either instantaneous (continuous time) or periodic (discrete time). This ideology has well served the field for 50 years. But the new demonstrations had shown that basic input/output signals are data packets that may arrive at variable times, not necessarily in order, and sometimes not at all. Networks between sensors, actuation, and computation must be taken into account, and algorithms must address the tradeoff between accuracy and computation time [52].

A networked control system is basically a feed back control systems wherein the control loops are closed through the real time network are called networked control systems(NCSs). The defining features of NCSs is that information (refrence input, plant output, control input etc.) is exchanged using a network among control systems components(Sensor, controller, actuator etc). The generic setup of NCS is as shown in fig. 1.1.

In other way it can be defined as, a control systems which comprised of the system to be controlled and of actuators, sensors, and controllers, the operation of which is coordinated via a shared communication network. These systems are typically spatially distributed, may operate in an asynchronous manner, but have their operation coordinated to achieve desired overall objectives.

Research on Networked control systems (NCS) has been the prime focus both in academia and in industrial applications for several decades. NCS has now developed into a multidisciplinary area. In this chapter, we provide an introduction to NCS and different advantages of having such systems. As we proceed further, the chapter gives an insight to different challenges faced with building efficient, stable and secure NCS. We also discuss the different fields and research arenas, which are part of NCS and which work together to deal with different NCS issues.

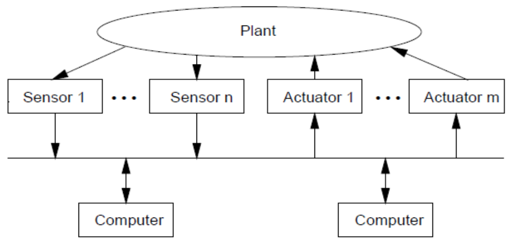

Figure 1.1: Generic setup of NCS

For many years researchers have given us precise and optimum control strategies emerging from classical control theory, starting from open-loop control to sophisticated control strategies based on genetic algorithms. The advent of communication networks, however, introduced the concept of remotely controlling a system, which gave birth to networked control systems (NCS). The classical definition of NCS can be as follows: When a traditional feedback control system is closed via a communication channel, which may be shared with other nodes outside the control system, then the control system is called an NCS.

1.2 Advantages of NCS

One of the biggest advantages of a system controlled over a network is scalability. As we talk about adding many sensors connected through the network at different locations, we can also have one or more actuators connected to one or more controllers through the network.

For many years now, researchers have given us precise and optimum control strategies emerging from classical control theory, starting from PID control, optimal control, adaptive control, robust control, intelligent control and many other advanced forms of these control algorithms. The use of a multipurpose shared network to connect spatially distributed elements results in flexible architectures and generally reduces installation and maintenance costs. Consequently, NCSs have been finding application in a broad range of areas such as in manufacturing plants, automobiles, and aircraft. Connecting the control system components in these applications, such as sensors, controllers, and actuators, via a network can effectively reduce the complexity of systems, with nominal economical investments. Furthermore, network controllers allow data to be shared efficiently. It is easy to fuse the global information to take intelligent decisions over a large physical space. They eliminate unnecessary wiring. It is easy to add more sensors, actuators and controllers with very little cost and without heavy structural changes to the whole system. Most importantly, they connect cyber space to physical space making task execution from a distance easily accessible (a form of tele-presence).

1.3 NCS Challenges

A challenging problem in the control of network-based systems is the network delay effects. The time to read a sensor measurement and to send a control signal to an actuator through the network depends on network characteristics such as topology and routing schemes.

NCS applications can be broadly categorized into two categories as

1. Time-sensitive applications or time-critical control such as military, space and navigation operations;

2. Time-insensitive or non-real-time control such as data storage, sensor data collection, e-mail, etc.

Network reliability is an important factor for both types of systems. The introduction of delays and packets dropouts in communication networks is inevitably, it is because of limited bandwidth, and also due to overhead in the communicating nodes and in the network. The delays in many systems will be varying in a random fashion. This makes the network unreliable and time-dependent. Therefore, the overall performance of an NCS can be affected significantly by network delays. The severity of the delay problem is aggravated when data loss occurs during a transmission. Moreover, the delays do not only degrade the performance of a network-based control system, but they also can destabilize the system.

1.4 Brief History of Research Field of NCS

As the concept of NCS started to grow because of its potential in various applications, it also provided many challenges for researchers to achieve reliable and efficient control. Thus the NCS area has been researched for decades and has given rise to many important research topics. A wide branch in the literature focuses on different control strategies and kinematics of the actuators/ vehicles suitable for NCS [1], [11], [6], [22], [95]. Another important research area concerning NCS is the study of the network structure required to provide a reliable, secured communication channel with enough bandwidth, and the development of data communication protocols for control systems [48], [49], [108]. Thus NCS is not only a multidisciplinary area closely affiliated with computer networking, communication, signal processing, robotics, information technology, and control theory, but it also puts all these together beautifully to achieve a single system which can efficiently work over a network.

1.5 Outline of the Thesis

The contents of the thesis are as follows:

Chapter 2: LITERATURE SURVEY

This chapter gives an detail overview of the recent work done in the area of Networked Control Systems. It summarizes the work done by various researchers in the field of Markovian jump control systems and model predictive based network control systems. The chapter is divided into sections, each section discussing a specific class of models based on the models of the network delay such as Constant delay, Random delay, which is independent from transfer to transfer and Random delay, with probability distributions governed by an underlying

Markov chain.

Chapter 3: MARKOV JUMP NETWORKED CONTROL SYSTEM-SWITH PACKETS DROPOUTS AND PARTIALLY KNOWN TRANSITION MATRIX

In this chapter, stability analysis and controller design of discrete Markov jump linear networked control system are studied.The focus of our study is on partially known transition matrices which relaxes the standard assumption on prior knowledge of these matrices. For a class of systems with partially known transition probabilities, feedback controller is designed and a sufficient condition for stochastic stability of the underlying system is derived via Linear matrix Inequality (LMI). The chapter is concluded with an example to illustrates the effectiveness of proposed methodology.

Chapter 4: NETWORKED PREDICTIVE CONTROL SYSTEMS WITH DYNAMIC OUTPUT FEEDBACK

In Chapter 4 we consider an NCS wherein stability analysis and controller design of networked control systems with packets dropouts are investigated using model predictive control approach. A novel dynamic output feedback is developed based on networked predictive control scheme by considering an important features of NCSs to transmit a packet of data set at the same time. The stability analysis for the proposed networked predictive control scheme is also investigated with partially known transition probability matrices.

Chapter 5: CONCLUSIONS AND FUTURE WORK

This chapter summarizes the main contributions of the thesis, provides very recent results in the area that the author became aware of by the time of completion of the thesis. Finally, suggestions for future work and developments are included in the last section of this chapter.

Chapter 2

LITERATURE SURVEY

2.1 Introduction

It is widely accepted that digital control systems were enhanced with communication networks around the year 1980. The rapid development in car industry was the driving force for the extensive use of communication network in digital systems. Reduced cost for cabling, modularization of systems, and flexibility in system setup were some of the prime motives in the introduction of communication networks. Technology advancement, together with performance and cost considerations, fueled the proliferation of networked control systems. In turn, many new fundamental questions were raised in communications, information processing, and control dealing with the relationship between operations of the network and the quality of the overall systems operation. Due to the low price of hardware devices for network and network nodes, control loops are being often closed over a communication networks and this trend is getting more and more common. Typically, a control system communicating with sensors and actuators over a communication network is called *a distributed real-time control system*. The general block diagram of NCS is as shown in fig.2.1. Network

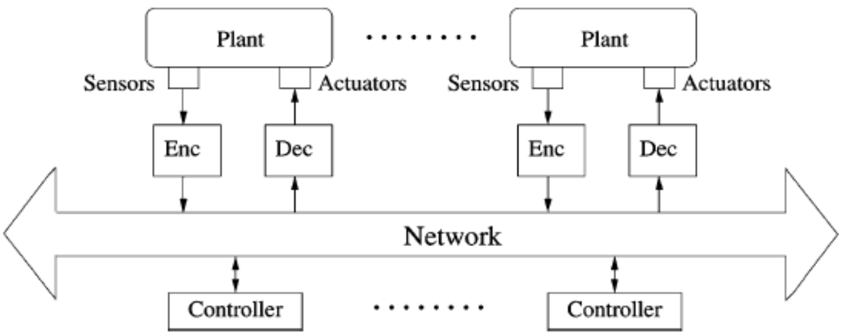

Figure 2.1: The General Block Diagram of NCS

nodes, actuator nodes, sensor nodes, and controller nodes are of specific interest in distributed control system. Process values are measured over sensor nodes and transmitted over the communication network. New values for the process inputs are received by actuator nodes over the communication network. Controller nodes read process values from sensor nodes. By using a control algorithm, control signals are calculated and transmitted to the actuator nodes via the data network. Clock-driven means that the node starts its activity at a pre specified time, for instance, the node can run periodically. Depending on how the sensor, actuator, and controller nodes are synchronized several setups can be found. Several previous authors have suggested control schemes with slightly different timing setups. The different setups come from whether a node is event-driven or clock-driven. By event-driven we mean that the node starts its activity when an event occurs, for instance, when it receives information from another node over the data network. Clock-driven means that the node starts its activity at a pre specified time, for instance, the node can run periodically.

Thus distributed control system is a powerful setup. It offers an advantage of modularity and flexibility in system design. But some care must be taken while handling it. The introduction of delays in communication networks is inevitably , it is because of limited bandwidth, and also due to overhead in the communicating nodes and in the network. The delays in many systems will be varying in a random fashion. From the perspective of control system, these varying delays will no longer be time-invariant. As an effect of this the standard computer control theory cannot be used in analysis and design of distributed real-time control systems. The introduction of communication networks in feedback control loops makes the NCS analysis and synthesis complex, where much attention has to be paid to the delayed data packets of an NCS due to network transmissions. In fact, data packets through networks suffer not only transmission delays, but also, possibly, transmission loss/packet dropout.

The computer delays are usually classified into three types:

- Communication delay between the sensor and the controller, d_k^{sc}.
- Computational delay in the controller, τ.

- Communication delay between the controller and the actuator, d_k^{ca}.

The control delay, τ_t for the control system, is the time from when a measurement signal is sampled to when it is used in the actuator, equals the sum of these delays, i.e., $\tau_t = d_k^{sc} + d_k^{ca} + \tau$

The delays in the networks are of different characteristics depending on the type of hardware and software used in the network.

Three models of the network delay which are generally considered:

1. Constant delay,
2. Random delay, which is independent from transfer to transfer,
3. Random delay, with probability distributions governed by an underlying Markov chain.

The problem of analysis and design of control systems when the communication delays are varying in a random fashion are mainly addressed in this thesis. The Markov chain model is the most advanced one that is used to generates the probability distributions of the time delays. Rigorous research has been carried out in this domain to ensure better efficiency and stability of Networked Control Systems.

A wide range of research has recently been reported dealing with problems related to the distributed characteristics and the effect of the digital network in networked control systems with the prime focus both in academia and in industrial applications. Only when the various network based phenomena came into picture, research on NCS progressed, and researchers tried to focus on the more practical aspects of Networked control. The earliest phenomenon to be studied in depth was the problem of transmission losses or packet losses. The literature review for this thesis incorporates recent surveys carried out by the various authors.

2.1.1 Network with constant delay

The simplest way to model the network delay is to model it as being constant for all transfers in the communication network. This type of model gives the satisfactory response, even if the network has delays which are varying with time, for example, if the time scale in the process is much larger when compared to the delay introduced by the communication channel. In such cases, the mean value or the worst case delay value is used in the analysis. Else if this is not the case, wrong conclusions can be drawn regarding system stability and performance. One of the simplest way to achieve constant delays is through the introduction of timed buffers after each transfer cycle. If these buffers are made longer than the worst case delay time the transfer time can be viewed as being constant. Several investigators have addressed the problems of delay compensation in closed loop control systems.

The research presented in [1], investigated the multivariable discrete systems with pure delays and derived the necessary and sufficient conditions for the optimization using nonlinear programming. It was shown that the principles developed can be applied any to systems with linear dynamics, convex inequality constraints, and convex performance index.

The authors of [11] investigated this problem in detail and were successful in eliminating the randomness of this time delays through the introduction of timed buffers. This, however, introduces extra time delay in the loop. A major drawback of this method is that the control delay becomes longer than necessary. This leads to decreased performance. The design for this scheme can be done in the same way as in the standard LQG-problem.

An intuitive approach was done in [6], where a new approach is suggested in which the delayed variables were augmented to system model as additional states. Unfortunately, this leads to some of the states as uncontrollable even when the original system is completely controllable.

For the case of delayed control inputs, a predictor for the optimal state trajectory is proposed based on past control inputs [2]. In [3], the problem of controlling a computer-controlled system with measurement and computational delays is studied. According to their findings the delays in multi-variable systems may result in

1. An increase in the magnitudes of the transients and poor response during the inter-sampling time,

2. Loss of decoupling between individual SISO control loops although decoupling may be restored for a stable process at the steady-state, and

3. A possible decrease in the stability margin. The algorithm was verified by simulation but the use of an observer to estimate the unavailable states was not discussed.

A significant amount of research work has been reported for observer and controller design in [7], [4], [8] for processes with inherent constant delays that occur within the process to be controlled.

In [7], a new decomposition approach is presented for linear discrete-time systems with delays. According to Drouin, through proper decomposition optimality conditions can be satisfied to obtains a control law with partial feedback and an open loop part. The obtained optimal control laws ensured a fast reaction with regard to small disturbances and of a correcting term which may be easily implemented. The use of such a law on online systems were also discussed in his work.

In [10], research investigators established that, a stable controller designed on the basis of a constant delay which is equal to the supremum of the varying delay may not ensure the system stability at all the time.

In [79], the published paper focusing on the problem of delay-dependent robust stochastic stability Markovian jump systems with state and input delays. The delays were considered to be constant and unknown, but assumed as bounded with the known limits. A new delay-dependent stochastic stability and stabilization conditions were proposed by constructing a new Lyapunov-Krasovskii functional and introducing some appropriate slack matrices in linear matrix inequalities (LMIs). An important conclusion made was, all stability and stabilization conditions are the function of the upper bound of the delays. Finally, a state feedback controllers was designed using the memory less system.

2.1.2 Network with independent delays

Network delays are usually random. The network delay can have several sources, for instance,

- waiting for the network to become idle,
- If several messages are pending, the wait can include transmission of the waiting messages,
- If transmission errors occur, a retransmission may be needed,
- In some networks collisions can occur if two nodes try to send at the same time, the resolution of this can include a random wait to avoid a collision at the next try.

Activities in the system are usually not synchronized with each other, as a result, the above listed delays takes place in random fashion. To take this randomness of the network delays into consideration of model, the time delays can be modeled as being governed by specific probabilistic distribution. To make the model simple in analysis, one can consider that the transfer delay are independent of previous delay times. In a real communication system the transfer time will, however, usually be correlated with the last transfer delay.

These delays which are independent from transfer to transfer, constitutes a Markov model with only one state.

In [14], the authors discussed the stability analysis and controller design of linear systems subjected to random jumps. Firstly, the mean square stability criteria is given for the case where the system is driven by the sequence of independent stationary random delays. Later the condition for almost sure stability was discussed with noise free case. The work done by Stanley lee in the area of NCSs with consideration of random delay is the basic foundation being provided for further research and analysis.

The authors of [16, 15] discussed in detail the effects of these random delays on the system and surveyed a few networked control techniques to be used in an unstable NCSs. The techniques proposed by them were straight forward, simple and easy. In addition, this methodology can be modified to support non-identical sampling periods.

The authors of [22] gave a review of some previous work on networked control systems (NCSs) and proposed future improvements. Fundamental issues in

NCSs were summarized. The effect of different underlying network-scheduling protocols was examined. Methods to compensate network-induced delay and experimental results over a physical network were discussed. Stability analysis was carried out for different NCSs models with packet dropout and transmission error.

In [24], the problem of the jump systems with time delay is addressed and proposed H_∞ disturbance attenuation controller. The delay-dependent sufficient conditions on the stochastic stability were presented. The main focus of the author's was to estimate the upper bound on the time delay, such that the jump system is stable. The proposed technique was very well extended to the case of multiple delay cases.

Lanzhi Teng in [55], developed a new model for a networked PID control system. The unreliable link of the shared networks was modeled using Markov chain. An embedded Markov chain was used to identify the minimum measurement information involved in the PID control system. The resultant was a jump control system.

In the same year, Dragan B in [56], published a paper where the stability conditions of linear networked control systems via simultaneous protocol and controller design was studied in detail. A simultaneous design of controllers and protocols were developed in terms of matrix inequalities conditions. The protocols obtained requires no knowledge of controller and plant states but only the current and the most recently transmitted values of signals nodes which are being applied on the controller area network.

A new controller design, where the controller uses event and time driving strategy was introduced in [77]. The state observer designed given by them was used not only for estimating the states but also for the compensation for time delay and dropout. The resultant closed system obtained is modeled as asynchronous dynamical system constrained by configuration event rate. Finally exponential stability is analyzed for this system. The experimental results shows that proposed methodology is validity for uncertain network-induced time-delay and data packet losses, and as well guarantees the stability of networked control systems with .

In [93], modeling of One-Dimensional Multi-Hop Network was carried out, where opportunistic array transmission was modeled as a Markov chain in discrete time. The Perron-Frobenius eigenvalue matrix was used in determining different parameters for achieving better performance in delivering the message to a destination.

A note published in [92], studied about the different mechanisms to reduce the packet loss rate and network induced delay. One of these mechanisms, namely, buffer/queue management was presented in detail. It was shown that techniques from robust control of uncertain time delay systems can be used effectively in combination with the above mentioned mechanism.

2.1.3 Network with Markov chains

Markov analysis is a powerful modeling and analysis technique with strong applications in time-based reliability and availability analysis. The reliability behavior of a system is represented using a state-transition diagram, which consists of a set of discrete states that the system can be in, and defines the speed at which transitions between those states take place. Markov models consist of comprehensive representations of possible chains of events, i.e. transitions within systems which, in the case of reliability and availability analysis, correspond to sequences of failures and repair.

The Markov model is analysed in order to determine such measures as the probability of being in a given state at a given point in time, the amount of time a system is expected to spend in a given state, as well as the expected number of transitions between states: for instance representing the number of failures and repairs.

To model phenomena as network queues, and varying network loads, our network model needs to have a memory, or a state. One way to model dependence between samples is by letting the distribution of the network delays be governed by the state of an underlying Markov chain. Effects such as varying network load can be modeled by making the Markov chain do a transition every time a transfer is done in the communication network.

As shown in fig.2.2, the process starts in one of the states and moves successively from one state to another. Each move is called a step. If the chain is currently in state s_i, then it moves to state s_j at the next step with a probability denoted by p_{ij} , and this probability does not depend upon which states the chain was in before the current state.

The probabilities p_{ij} are called transition probabilities. The process can remain in the state it is in, and this occurs with probability p_{ii}. An initial probability distribution, defined on S, specifies the starting state. Usually this is done by specifying a particular state as the starting state.

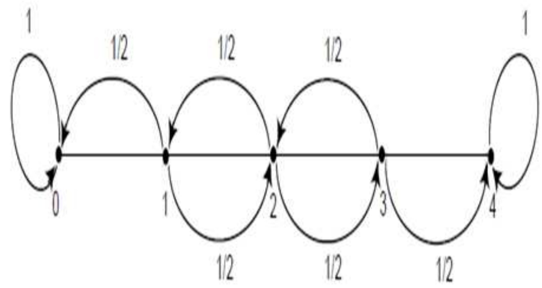

Figure 2.2: State-transition diagram

$$\rho = \begin{bmatrix} \rho_{11} & \rho_{12} \cdots \rho_{1N} \\ \rho_{21} & \rho_{22} \cdots \rho_{2N} \\ \\ \rho_{N1} & \rho_{N2} \cdots \rho_{NN} \end{bmatrix}$$

$$\lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \cdots \lambda_{1N} \\ \lambda_{21} & \lambda_{22} \cdots \lambda_{2N} \\ \\ \lambda_{N1} & \lambda_{N2} \cdots \lambda_{NN} \end{bmatrix}$$

The entries in the first row of the matrix ρ and λ represent the probabilities for the various kinds of delay that can exist in measurement channel(S/C) and actuation channel(C/A) respectively starting from initial condition during the first transition cycle. Similarly, the entries in the second and third rows represent the probabilities for the various kinds of delays in following transition cycle. Such a square array is called the matrix of transition probabilities, or the transition matrix.

For an instance, consider a system with three modes. i.e, the delay in the measurement channel can take any one of these values $S = \{0, 1, 2\}$. Let the matrix ρ_{ij} be,

$$\rho = \begin{bmatrix} .4 & .3 & .1 \\ .8 & .2 & 0 \\ .22 & .55 & .23 \end{bmatrix}$$

From the above given matrices, we see that if their is a delay of **zero** at first transition cycle then the event that their would be a delay of **two** from now is the disjoint union of the following three events 1) Their is a delay of **zero** at next transition cycle and a delay of **two** after two transition cycle from now 2) Their is a delay of **one** at next transition cycle and a delay of **two** after two transition cycle from now, and 3) Their is a delay of **two** at next transition cycle and also after two transition cycle from now. The probability of the first of these events is the product of the conditional probability that their is a delay of **zero** at next transition cycle, given that their is a delay of **zero** at current instant, and the conditional probability that their is a delay of **two** after two transition cycle from now, given that their is a delay of **zero** at next transition cycle. Using the transition matrix ρ, we can write this product as $\rho_{11}\rho_{13}$.

The other two events also have probabilities that can be written as products of entries of ρ. Thus, we have

$$\rho_{13} = \rho_{11}\rho_{13} + \rho_{12}\rho_{23} + \rho_{13}\rho_{33}$$

This equation is similar to the dot product of two vectors, where the first row of ρ is dotted with the third column of ρ.

The model is closely related to the models used in jump systems. A difference is that in our network model each state of the Markov chain postulates probability distributions for d_k^{sc} and d_k^{ca} . *In jump linear system each Markov state defines a set of system matrices,* $\{A(r_k), B(r_k), C(r_k),$
$D(r_k)\}$. It can also be noticed that in the previously discussed model, where delays are independent from transfer to transfer, constitutes a Markov model with only one state.

2.1.4 Linear systems over Markov communication channels

- **Packet Dropout in the Measurement Channel(S/C)**

  Much research has been directed towards the study of effect of communication packet losses in the feedback loop of a control system. For such problems, a simple packet-loss model was considered for the communicated information and the results for discrete-time linear systems with Markovian jumping parameters was applied. The goal was to find a controller (if one exists) such that the closed loop is mean square stable for a given packet loss rate. A linear matrix inequality (LMI) condition is developed for the existence of a stabilizing dynamic feedback controller.

The authors in [18],[12] and [9] thoroughly investigated the networked control systems where the delay in the channel(S/C) is modeled Using Markov Chain. They considered a plant, where the states of the network are modeled by a Markov chain and Lyapunov equations was used to derive the expected LQG performance. An illustrative examples were also presented which supported their proposed theory. The model used by author's in the analysis is as shown in fig.2.3

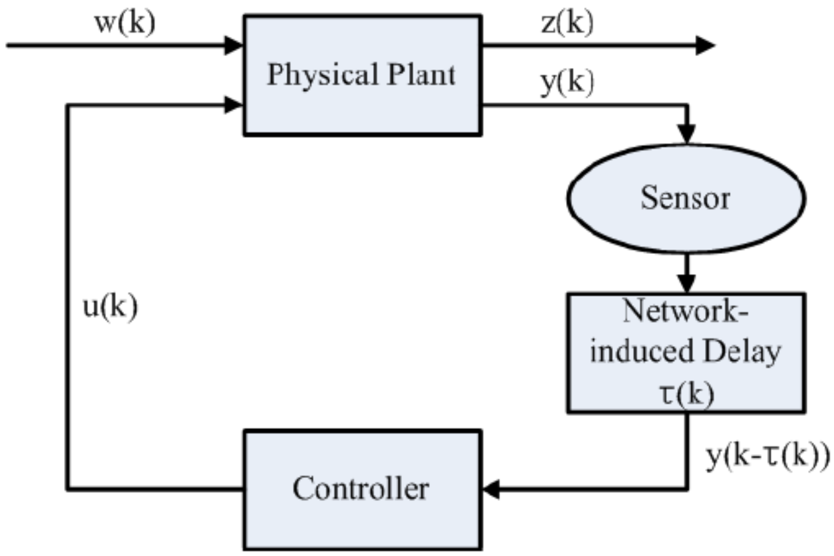

Figure 2.3: The setup of Network Control System with delay in only measurement channel

Authors in [32] were among the first few to model the delay in the networks using Markov chain. Their work was mainly focused on control of continuous-time systems were randomly varying delay was parametrized by finite-state Markov processes. They examined the Controllability, stabilizability properties in infinite time jump linear quadratic (JLQ) optimal. Assuming that the solution of the continuous-time Markov JLQ problem with finite or infinite time horizons is known, their work mainly focused on the sufficient conditions for the existence of finite cost and stabilizing controls.

In [31], authors considered an equation for a networked control system's performance as a function of the network's dropout process. The dropout process was governed by a Markov chain. The equation computes the system output power as a function of the Markov chain's transition matrix. The results in this paper extend earlier results in [26] to multi-input multi-output systems as well as to a more general class of dropout processes.

The effect of communication packet losses which occur in various vehicle control problems was investigated in [21] by P. Seiler and R. Sengupta. By considering a simple network model, they were successfully able to apply results for discrete-time MJLS. An LMI condition was developed for the existence of a stabilizing dynamic output feedback controller. But the vehicle following example shows that these results need to be extended to measure performance versus packet loss rate. Furthermore, vehicle following problem has the rather naive set up that each vehicle uses only information from its predecessor.

With similar results and analysis in [13], a digital control systems is modeled with random but bounded delays in the feedback loop as finite-dimensional, discrete-time jump linear systems, with the transition jumps being modeled as finite-state Markov chains. This type of system can be called a stochastic hybrid system. Due to the structure of the augmented state-space model, control of such a system is an output feedback problem, even if a state feedback law is intended for the original system. They proposed a V -K iteration algorithm to design switching and non-switching controllers for such systems. This algorithm involves an iterative process that requires the solution of a convex optimization problem constrained by linear matrix inequalities at each step.

Authors in [40], proposed an iterative approach to model networked control systems (NCSs) with arbitrary but finite data packet dropout as switched linear systems. Sufficient conditions were presented on the stability of NCSs with packet dropout and network delays. The basic idea behind this approach is that, the controllers can make full use of the previous information to stabilize NCSs when the current state measurements are lost during the course of transmission over the network.

In [46], authors studied a class of linear continuous-time systems with stochastic jumps were the jumping parameters are modeled using a continuous-time, discrete-state homogeneous Markov process. Sufficient conditions for the stochastic stability of the underlying system was proposed using Linear matrix inequalities (LMIs) approach. It has been shown that for the Markovian jump systems, the proposed sliding mode control is solvable only if a set of coupled LMIs have solutions. A numerical example was also presented to show the effectiveness of the proposed controller.

A controller was designed for Markov jumping systems in [47] with actuator saturation. The closed-loop stability was analyzed using the criteria of domain of attraction in mean square sense. A mode-dependent state feedback controller is developed only when the jumping mode is available, Else other approach is used to design the mode-independent state feedback controller. Linear matrix inequalities (LMIs) was used in design procedures of both the controllers.

In [63] authors studied the problem of stabilization for networked stochastic systems with transmitted data dropout. A discrete stochastic time-delay nonlinear system was considered and modeled using TakagiSugeno fuzzy method. Lyapunov function and a fuzzy Lyapunov function were used to develop the exponential stability criteria of NCSs. Finally, three numerical examples were presented to demonstrate the potential of the proposed technique.

Another paper was published with similar results in [97], in which the problem of stability analysis and control synthesis for Markovian jump linear systems with time delays and norm-bounded uncertainties was studied. The model with different time-invariant discrete, neutral and distributed delays was considered and a mode-dependent delayed state feedback H_∞ controller was designed in terms of linear matrix inequalities (LMIs). The designed controller guaranteed the stochastic stability for the closed-loop system under consideration.

Ligang Wu et al.[95] proposed a new and necessary condition sufficient for the stability in terms of linear matrix inequality (LMI). Their work was mainly concerned with the state estimation and sliding mode control problems for continuous-time Markovian jump singular systems with unmeasured states. In order to estimate the system states, observer was designed, and based on these estimated states sliding mode controller was designed. It was shown that within the finite time sliding mode estimation space can be attained.

- **Packet Dropout in the Measurement (S/C) and Actuation (C/A) Channels**

  It is noticed that all the stability conditions and controller designs given in the aforementioned references are derived based on the assumption that packet dropout exists only in the sensor-to-controller (S/C) side. The effect of controller-to-actuator (C/A) packet dropouts is neglected due to the complicated NCSs modeling.

  Recently, some results were obtained in [44][34][20] were NCSs with packet dropouts on both S/C and C/A sides are considered.

  The analysis is carried out in [80] by considering the packet dropouts in both measurement and actuation channel. The work was mainly concerned with the robust H_∞ output feedback control for a class of uncertain discrete-time delayed nonlinear stochastic systems. The time-varying delay was assumed to be unknown with known lower and upper bounds. Lyapunov method and stochastic analysis techniques were employed in deriving the sufficient condition for the existence stability. LMI tool was used for obtaining the controller parameters.

In [48], investigation was done for a class of uncertain neutral stochastic time-varying delay systems. To reduce the impact of the input disturbance on the controlled output up to a acceptable level, a robust stochastic stabilization technique was proposed, in which the mean-square asymptotic stability of the resulting closed-loop system is defined using the set of LMI's. A special case of time delays appearing in both the state and control input was discussed and the necessary controller was designed for it.

In [65], stochastic stability of networked control is established with both network induced delay and data dropouts Lyapunov approach was used to derive the sufficient conditions for the mean-square stability of the networked control system. Linear matrix inequalities(LMIs) were used to construct the stabilization controllers.

In [87], authors considered the problem of stabilization for a networked control system with Markovian characteristics. Author's in this paper considered that, the delay existed both in the measurement channel and actuation channel which are modeled as a Markov chain. The resulting closed-loop system obtained was Markovian jump linear system with two modes. Linear matrix inequality (LMI) technique was used in obtaining the controller parameters and necessary stability conditions.

Researchers in [49], introduced a new stabilization technique for a class of linear jump systems by using the concept of zero equation. According to zero equations concept, the delay-dependent results are independent of model transformation and bounds on the cross terms. Robust stabilization condition were provided by corresponding H_∞ control law with the allowance of prescribed H_∞ disturbance attenuation level.

NCS with similar consideration was studied by Wu-Hua Chen et al.[39]. In this, author's have proposed a cost guaranteed control problem for a class of continuous-time Markovian jump linear system. A state feedback control law was proposed such that the resulting closed-loop system has stochastically stable behavior and the cost value function of this closed-loop is limited to upper bound. It was shown that stability conditions are function of upper and the lower time-delays bounds. A new concept of Moon's inequality was presented to derive the sufficient conditions for the existence of controller and to guarantee the stability.

In [82], optimal Kalman filtering problem of the network control systems subjected to random sensor delay, missing data and packet dropout was analysed in detail. Markov chain, was used to accommodate random delay, packet dropouts and missing measurements. In their analysis, the gain of the optimal filter was computed offline in advance. An examples was presented to illustrate the effectiveness of the proposed methodology.

In [83], output feedback stabilization of discrete-time jump linear networked control systems (NCSs) was investigated. The sensor-to-controller (S-C) and controller-to-actuator (C-A) random network-induced delays are modeled as Markov chains. The focus is on the design of a two-mode-dependent controller that depends on not only the current S-C delay but also the most recent available C-A delay at the controller node. The block diagram of the NCS considered in the analysis is as shown in fig.2.4

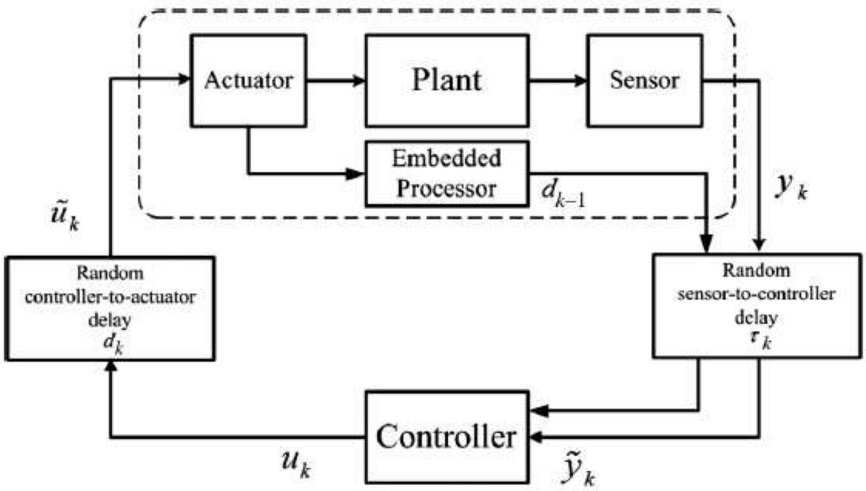

Figure 2.4: Diagram of a networked control system.

In [61], Network control system with data packet dropouts and transmission time delays is considered. A two-state Markov chain is used to model the time-varying and uncertain, the data packet dropout. The concept of compensator is established to compensate the lost packet, a data packet dropout. Sufficient conditions for the stabilization of the new resulting system are derived in the form of linear matrix inequalities (LMIs). The conditions concerned with the delay and data packet dropout for the preservation of the NCSs stability were based on Lyapunov theory.

In [107], network-based H_∞ filtering for discrete-time systems is presented. Network-induced delays and packet dropouts caused due to a constraint communication network are studied in detail. A new approach is studied in which, logic data packet processor chooses the newest data signal from the network to actuate the filter. As a result, the designed is modeled as a Markov jumping linear filter. Stochastic stability with a prescribed H_∞ level is derived using bounded real lemma (BRL).

In [108], different models of NCS were presented and robust asymptotic stability and controller design was investigated. The first model presented was of NCS with single mode Markov chain and external disturbance. A nonlinear model of NCS was also analysed by applying cerebellum model articulation controller(CMAC). These models describe the bounded delay, packet dropouts and disorders.

Jing Wu and Tongwen Chen in [59], discussed about the linear/nonlinear LQG optimality condition to minimize the cost function according to the transmission control protocol (TCP) and user data-gram protocol (UDP). In their note, Markov chains was introduced to describe S/C and C/A packet dropouts. The Markov chains was considered to describe the quantity of packet dropouts between current time k and its latest successful transmission instead of only the information on if a packet is dropped or not, which is different from the previously mentioned works. By this definition, the number of states of Markov chains is larger than two and the history of packet dropouts can be seen clearly.

The discrete linear time-invariant plant model used is as shown in fig(2.5)

Consider the plant model as:

$$x(k+1) = \phi x(k) + \Gamma u(k) \tag{2.1}$$

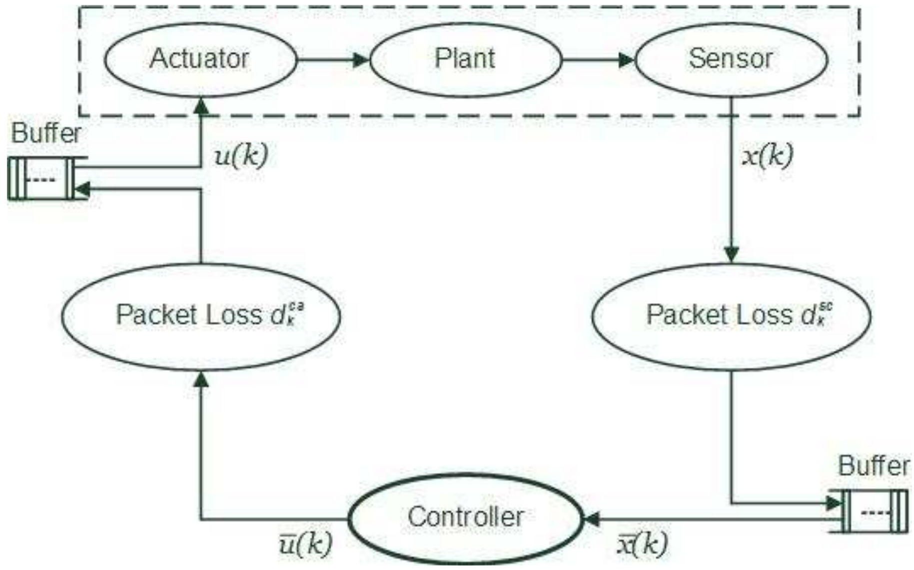

Figure 2.5: The setup of Network Control System

where $x(k) \in \Re^n$ is the plant state vector and $u(k) \in \Re^m$ and ϕ and Γ are known real constant matrices with appropriate dimensions. The buffers are supposed to be long enough to hold all the packets arrived, which will be picked up according to the last-in-first-out rule. For example, during the transmission from sensor to controller, the sensor data $x(k)$ is lost, the controller will read out the most recent data $x(k-1)$ from the buffer and utilize it as $\bar{x}(k)$ to calculate the new control input, which will be sent to the plant; otherwise, the new sensor data $x(k)$ will be saved to the buffer and used by the controller as $\bar{x}(k)$. Thus, for the buffers, we have

$$u(k) = \begin{cases} \bar{u}(k) & \text{if successfully transmitted} \\ u(k-1) & \text{otherwise} \end{cases} \tag{2.2}$$

$$\bar{x}(k) = \begin{cases} x(k) & \text{if successfully transmitted} \\ \bar{x}(k-1) & \text{otherwise} \end{cases} \tag{2.3}$$

The quantity of packets dropped at time k on the S/C(sensor-controller) side is assume to be d_k^{sc}, which is calculated from the current time k to the last successful transmission (happened at time $k - d_k^{sc}$), d_k^{ca} is the packet quantity dropped on the C/A(controller-actuator) side between the current time k and its last successful transmission at time $k - d_k^{ca}$, and both of them are bounded. Thus, we have

$$0 \leq d_k^{sc} \leq d_1, \;\; 0 \leq d_k^{ca} \leq d_2$$

where d_1 and d_2 are nonnegative integers. d_k^{sc} and d_k^{ca} are modeled as two homogeneous independent Markov chains, which take values in $S_1 = \{0, 1, \cdots, d_1\}$ and $S_2 = \{0, 1, \cdots, d_2\}$.

The transition probabilities matrices are defined by

$$\rho_{ij} = Pr(d_{k+1}^{sc} = j | d_k^{sc} = i)$$
$$\lambda_{mn} = Pr(d_{k+1}^{ca} = n | d_k^{ca} = m) \tag{2.4}$$

Where $\rho_{ij} = 0, i, j \in S_1, \lambda_{mn} \geq 0, m, n \in S_2$ and $\sum_{j=0}^{d_1} \rho_{ij} = 1, \sum_{n=0}^{d_2} \lambda_{mn} = 1$. It is obvious that the transition probabilities satisfy

$$\rho_{ij} \geq 0, \quad if \;\; j \neq i+1 \;\; and \;\; j \neq 0$$
$$\lambda_{ij} \geq 0, \quad if \;\; n \neq m+1 \;\; and \;\; n \neq 0$$

Consider the state feedback control law as

$$\bar{u}(k) = F(d_k^{sc})\bar{x}(k) \tag{2.5}$$

where $F(d_k^{sc})$ is a set of controllers and will be designed based on d_k^{sc}. Substituting (2.2) and $\bar{u}(k)$ into the system in (2.1), we get the following closed-loop system:

$$x(k+1) = \begin{cases} \phi x(k) + \Gamma F(d_k^{sc})\bar{x}(k) & \text{if } d_k^{ca} = 0 \\ \\ \phi x(k) + \Gamma u(k-1) & \text{otherwise } d_k^{ca} > 0 \end{cases} \tag{2.6}$$

Note that $\bar{x}(k) = x(k - d_k^{sc})$. In order to simplify the expression of the closed-loop system, we introduce a function $\alpha(\cdot)$ to combine the previous closed-loop system as

$$\begin{aligned} x(k+1) &= \phi x(k) + \alpha(d_k^{ca})\Gamma u(k-1) \\ &+ [1 - \alpha(d_k^{ca})]\Gamma F(d_k^{sc}) x(k - d_k^{sc}) \end{aligned} \tag{2.7}$$

$$\begin{aligned} u(k) &= \alpha(d_k^{ca}) u(k-1) \\ &+ [1 - \alpha(d_k^{ca})] F(d_k^{sc}) x(k - d_k^{sc}) \end{aligned} \tag{2.8}$$

where

$$\alpha(d_k^{ca}) = \begin{cases} 1 & d_k^{ca} > 0 \\ \\ 0 & d_k^{ca} = 0 \end{cases}$$

By the Concatenation of plant and controller state vectors we obtain a global vector $z(k) = [x^T(k)\ u^T(k-1)]$. Thus the closed-loop system obtained for the NCSs with single-packet transmissions is

$$\begin{aligned} z(k+1) &= \begin{bmatrix} \phi & \Gamma\alpha(d_k^{ca}) \\ 0 & \alpha(d_k^{ca}) \end{bmatrix} \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix} \\ &+ \begin{bmatrix} (1-\alpha(d_k^{ca}))\Gamma F(d_k^{sc}) & 0 \\ (1-\alpha(d_k^{ca}))F(d_k^{sc}) & 0 \end{bmatrix} \begin{bmatrix} x(k-d_k^{sc}) \\ u(k-d_k^{sc}-1) \end{bmatrix} \end{aligned}$$

$$z(k+1) = A(d_k^{ca})z(k) + B(d_k^{sc}, d_k^{ca})z(k-d_k^{sc}) \tag{2.9}$$

Therefore the resulting closed-loop system is a Markovian jump linear system(MJLs) with two modes (d_k^{sc} and d_k^{ca}) and one mode-dependent time-varying delay d_k^{sc} , where their transitions are described by two Markov chains, which give the history behavior of S/C and C/A packet dropouts, respectively.

This paper [59] was mainly concerned with the stability analysis and controller design of networked control systems(NCSs) with packets dropouts with known transition matrix. The work was based on the assumption that all the elements of the transition matrices ρ_{ij} and λ_{ij} are completely known in advance. The issue of partially known matrices was completely ignored by the authors. The work presented in the chapter **3** is an extension of the analysis carried out in [59] by taking into consideration partially known matrices that effects the practical working of networked systems.

2.1.5 Markov channels with partially known transition probability matrices

In Markov systems, the transition probabilities of the jumping process are crucial and all of them must be completely known a priori. However, the likelihood to obtain the complete knowledge on the transition probabilities is questionable and the cost is probably high. Take VTOL (vertical take-off landing) helicopter system in the aerospace industry for example, the air speeds variation involved in the system matrices are modeled as a Markov Chain. However, not all the probabilities of the jumps among multiple air speeds are easy to measure. In fact, from 135 knots (normal value) to 135 knots (dwell in one mode), may obtain the accurate probability or estimate a scope (uncertain one) effortlessly, but for the cases from 135 knots to any of 60, 70 or 80 knots, for instance, the probability is likely not accurate and the uncertainties bounds for them are quite ideal. The same problems may arise in other practical systems with Markovian jumps.

Thus, it was significant and challenging to further study more general jump systems with partially known transition probabilities from control perspectives, especially with time-varying delays included. More recently, some attention have been already drawn to the class of systems with time delays for both continuous-time and discrete-time.

In [60] authors focussed on the issue related to the stability of network control systems with packets loss. Packet-losses were modeled using two different processes. Firstly through the arbitrary process, the second was underlying Markovian chain process, where transition probabilities are partially known. Packet-loss dependent Lyapunov approach was used to establish the stability conditions of networked control systems with these two packet loss processes. Later it was shown that the same results can also applied to the unit time delay case Lyapunov technique was used in the controller design.

In [73], the problem of NCS with uncertain, time-varying network-induced delays and a bounded number of subsequent packet dropouts was studied in detail. The stability analysis and controller synthesis problem was proposed based on LMI conditions. The two following controllers were developed

1. feedback controller (that depends on both the state and the past control inputs) and
2. state-feedback controller

Performance analysis of both controllers is compared in terms of a lower bound for the transient decay rate of the response. The results are illustrated by application to a typical motion control example.

A fault detection problem for a class of discrete-time Markov jump linear system (MJLS was developed in [64], where transition probabilities are partially known. A fault detection and isolation(FDI) scheme is used and detected fault is used in the formulation of H_∞ filtering by which the error between residual and fault is minimized. The proposed method is more general than the traditional MJLS, where all the transition probabilities are completely known. The conditions of FDI filter is obtained based on LMI. Through simulations on an illustrative examples, it was shown that a trade off existed between the cost of obtaining the transition probabilities and the time taken to detect the faulty condition.

L.Zang et al.[81], published a paper on similar lines and performed the analysis of class of discrete-time Markov jump linear systems (MJLS) with partially known transition probabilities. In their study, a reduced-order model was considered and it was demonstrated that model error reduces to zero and is internally stochastically stable and has a guaranteed H_∞ performance index by suitable selection of LMI based conditions .

In [62], authors described the stability analysis and stabilization problems for a class of discrete-time Markov jump linear systems with partially known transition probabilities and time-varying delays are investigated. The time-delay is considered to be time-varying and has a lower and upper bounds. The transition probabilities of the mode jumps are considered to be partially known, which relax the traditional assumption in Markov jump systems that all of them must be completely known a priori.

In [78], authors, studied in detail the problem concerning the H_∞ estimation for a class of Markov jump linear systems (MJLS) with varying transition probabilities (TPs) in discrete-time domain. Two types of variations were considered in the finite set of time-varying TPs: arbitrary variation and stochastic variation. Stochastic variation means that, variation is subject to a higher-level. The closed-loop systems is made to be stochastically stable by designing variation-dependent H_∞ filter.

A class of extended continuous-time Markov jump linear systems was proposed by L.Zang and E.Boukas in [86]. The class of discrete dynamic systems is described by a Markov stochastic process, but with only partially known transition probabilities. The uncertain transition probabilities presented in their work requires no structure (polytopic ones), bounds (norm-bounded ones) or nominal terms (both) for the partially unknown elements in the transition rate matrix. The sufficient conditions for H_∞ control are derived via the linear matrix inequality formulation such that the closed-loop system is stochastically stable and has a guaranteed H_∞ noise-attenuation performance. A kind of tradeoff was seen between the difficulties to obtain all the transition probabilities and the systems performance benefits.Transition probabilities matrices, ρ and λ in this paper were considered as,

$$\rho = \begin{bmatrix} \rho_{11} & \rho_{12} & ? \\ ? & \rho_{22} & \rho_{32} \\ \rho_{31} & ? & ? \end{bmatrix}$$

$$\lambda = \begin{bmatrix} ? & \lambda_{12} & ? \\ ? & ? & \lambda_{23} \\ ? & ? & ? \end{bmatrix}$$

where the unknown elements are represented by "?". For notation clarity, $\forall i \in \mathcal{I} = \{1, 2, \cdots\}$, we denote

$\mathcal{I}_{\mathcal{K}}^{i} \triangleq \{j$:if ρ_{ij} is known$\}$, $\mathcal{I}_{u\mathcal{K}}^{i} \triangleq \{j$:if ρ_{ij} is unknown $\}$

$\mathcal{I}_{\mathcal{K}}^{m} \triangleq \{n$:if λ_{mn} is known $\}$, $\mathcal{I}_{u\mathcal{K}}^{m} \triangleq \{n$:if λ_{mn} is unknown$\}$

In [96], a new approach was explored for the analysis and synthesis for Markov jump linear systems with partially known transition. Transition rate matrices (TRMs) in continuous-time domain, or transition probability matrices (TPMs) in discrete-time domain were studied in detail. Sufficient conditions for the stability and stabilization are obtained for both the domains and supported fully by an illustrative example.

Authors of [101] examined the robust partially mode delay dependent H_∞ output feedback controller design for discrete-time systems with random communication delays. A finite state Markov chain with partially known transition probabilities is used to model random communication delays between sensors and controller. Based on Lyapunov-Krasovskii functional, a novel methodology for designing a partially mode delay-dependent output feedback controller was proposed.

In [98], a class of continuous-time Markovian jump systems with partial information on transition probability is considered. A new method was proposed, namely, free-connection weighting matrix which reduced conservatism on stability criteria. As a result, linear matrix inequalities technique can be easily applied to obtain the sufficient condition for the state feedback controller.

In [103], a packet-based NCS control approach is proposed. According to this approach, control law can be designed with explicit compensation for the network-induced delay, data packet dropout and data packet disorder in both forward and backward channels. As a consequence, this approach offers the designers the freedom of designing different controllers for different network conditions. The communication constraints are modeled as a homogeneous ergodic Markov chain.

In [100], The sufficient conditions for mean square exponentially stabilization of the closed loop system was given by using the Markov theory and the Average Dwell Time (ADT) method. By solving a series of linear matrix inequalities (LMIs) state feedback controller is designed.

Marco H. et al.[105] proposed a new control approach to regulate discrete-time Markovian jump linear systems subjected to partially known parameters. Standard LQR and the robust penalty game approach was used to solve the problem of DMJLS subjected to uncertainties.

In [111], output-feedback fault tolerant control problem for networked control system (NCS) is investigated. The total time-varying and unknown network-induced delay in both sensor-to-controller link and controller-to-actuator link is separated into two parts,

1. constant
2. bounded variable

The data packet dropout is modeled as two switches that follow a four-state Markov chain. Then a discrete-time Markovian jump uncertain model of NCS with actuator (complete/partial) failure is established.

2.2 Model Predictive Control For Networked Control Systems

Model predictive control (MPC), also known as moving horizon control or receding horizon control, has received much attention in the past decades due to its extensive applications in the control of industrial processes such as distillation and oil fractionation, pulp and paper processing. The basic principle of MPC controller is to predict the change in the dependent variables($y(k)$) of the modeled system that will be caused by changes in the independent variables($u(k)$). MPC uses the current plant measurements($x(k|k)$), the current dynamic state of the process, the MPC models, and the process variable targets and limits to calculate future changes in the dependent variables. These changes are calculated to hold the dependent variables close to target while honoring constraints on both independent and dependent variables. It works on the principle of receding horizon control, where only the first change in each independent variable($u(k|k)$) is implemented, and the calculation is repeated when the next change is required. This feature renders the MPC approach very appropriate to incorporate the input/output constraints into the on-line optimization as well as to compensate time delays, which increases the possibility of its application in the synthesis and analysis of NCSs.

In the past, much research has been done in the area of implementation of MPC in the synthesis and analysis of NCSs.

In [29], an article was presented for distributed model predictive control (MPC), focusing on

1. the coordination of the optimization computations using iterative exchange of information and

2. the stability of the closed-loop system when information is exchanged only after each iteration.

In [76] authors demonstrated Nonlinear Model Predictive Control (NMPC) in a framework of general event-based/asynchronous systems. It was shown that NCSs suffering from random delays and information losses can be made to work in a stable environment by proposed NMPC technique. The key to to guarantee asymptotic convergence, is the selection of large prediction horizon value. Authors were successful in developing a simple but effective algorithm to compensate random measurement, computation and actuation delays presented in networks. Some light was also shed on minimizing the amount of information exchanged between controller and actuator to reduce the data loss.

In [88], a deeper investigation was done on the stability of networked control systems through Nonlinear Model Predictive Control technique. A new approach called 'Set Invariance' was suggested. The work mainly deals with the constrained robust state feedback stabilization of uncertain discrete-time. Robust control policy was combined with model predictive control scheme to take into consideration various network defects like model uncertainty, time-varying transmission delays and and packet dropouts which effects the system stability and performance.

In [42], authors studied the plant state behavior between transmission times, and suggested an explicit model to estimate this plant states. Model-based networked control systems (MB-NCSs) were investigated in detail and stability condition was derived. Three feedback network communication models were presented in the paper. In the first model, Lyapunov technique was used to analyze unknown statistical properties of the update times. In the other two model the stochastic characteristics of the update time process was assumed to be known. These known stochastic characteristics were modeled using underlying Markov chain. Almost sure stability, and mean-square stability criteria was analyzed for each model.

G. P Liu et al.[53], came up with a modified MPC technique to handle the delays and dropouts in the measurement and actuation channel of NCSs. Predicted sequence of future control signal is used to compensate for the delay in the forward communication channel. Another important contribution made by this paper was to analyze the stability criteria for both, fixed and random communication time delays.

In [43], authors proposed a of model predictive control for network control systems with time stamping feature added to it. In their analysis they considered that the communication delays introduced by the networks are of random nature bounded with in minimum and maximum values. The proposed time-stamping with a buffer improves the performance and stability of the network over wider range of delay. Experimental validations were provided to support the proposed technique.

In [72], a compensator is proposed for network delay at the actuator using past control signals. They are suitable for a large number of networked control systems. In this scheme, control signals are always sent to the actuator within a fixed time interval, making the performance of the overall NCS network deterministic and predictable.

In [71], authors have considered networked control systems with random network delay in the forward channel and proposed H_2 a novel networked predictive controller to overcome the effects of network delay and data dropout. Time delays from sensor to controller and from controller to actuator are modeled as Markovian chains.

Authors of [51] presented a paper on neural network model predictive controller.The control law was represented by a neural network function approximate, which is trained to minimize a control-relevant cost function. The proposed procedure can be applied to construct controllers with arbitrary structures, such as optimal reduced-order controllers and decentralized controllers. A Numerical examples was presented to show that the neural network model predictive controller can be trained to achieve near-optimal control performance.

In [84], the prime focus was on the problem of time delays in network control system which can cause instability of closed loop operation of these systems. A state feedback controller is used and along with it, a predictive control scheme is implemented to design variable gains of the feedback controller depending on the number of packets missed (packet drop-outs) and time delays of the received input sample or state of the plant, both of which can be random but bounded for a given communication channel.

The work in [85] is concerned with the robust state feedback stabilization of uncertain discrete-time constrained nonlinear systems in which the loop is closed through a packet-based communication network. In order to cope with model uncertainty, time-varying transmission delays and packet dropouts which typically affect networked control systems, a robust control policy, which combines model predictive control with a network delay compensation strategy, is proposed.

In [91], a co-design approach was proposed for both a network and controller, a new predictive compensation method for variable delay and packet loss in NCS is designed, where simultaneous end-to-end delay and packet losses during transmission from sensor to actuator is tackled.

In [109], author proposed a decentralized MPC, exploring the trade-off which is inherited with the decentralization procedure and proposed sufficient criteria to test closed-loop stability a-posteriori. Two frameworks were considered, for open-loop stable systems, where the controller's task is performance optimization, and for unconstrained stabilizable plants.

In [106], an explicit model predictive controller was proposed to estimate the packet dropout in Networked Control Systems (NCS). In his work, model for networked control systems with data packet dropout is firstly transformed into piecewise affine form. Then constrained optimal control problem with PWA model is solved. By using the method of explicit predictive control, which is based on multi-parametric quadratic programming method, State feedback control law is obtained which is explicit to the states. Since the algorithm does not require repeated line optimization, it improves the on-line calculation speed of the controller, making the system better real-time. This algorithm also deals with the data packet dropout as well.

In [110],a Lyapunov- based method is employed to deal with the estimator design problem. When the delay is not large, the system with delayed state can be transformed into delay-free systems. By using the probabilistic delay model and the augmentation, the H_∞ filter design algorithm is proposed for networked systems. In order to compensate for the delays on both communication links, the predictive control scheme is adopted. To make full use of the delay information, it is better to use the Markov chain to model the network-induced delays and the missing packets.

Jing Wu et al.[70] proposed a Model predictive control for networked control systems. They designed a predictive control strategy for an NCS, such that at each sampling instant the infinite horizon quadratic objective is minimized while guaranteeing the stochastic stability of the closed-loop system. In their design consideration, actuator implemented the most recently received signal directly to the plant and only the first control move will be used. The networked communication delays are assumed to be random and bounded which are described by Markovian chains. Delay-dependent conditions for the existence of such controllers are given and a linear matrix inequality (LMI) approach was developed.

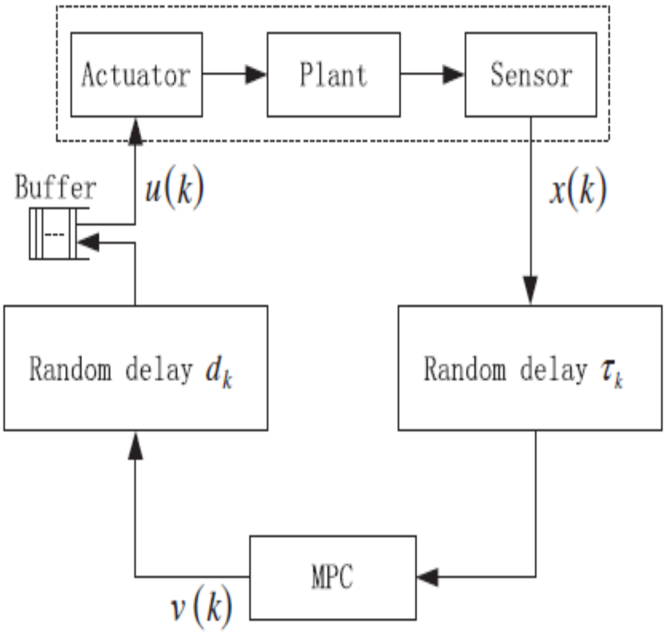

Figure 2.6: The setup of Network control system with MPC

The discrete linear time invariant model used in their analysis was:

$$x(k+1) = Ax(k) + Bu(k) \tag{2.10}$$

where $u(k)$ is the control input to the plant and $v(k)$ is output of MPC controller.

$v(k)$ satisfies the equation:

$$v(k) = F_k x(k - \tau_k) \tag{2.11}$$

If $\tau_k \geq 0$ is the random time delay from the sensor to the controller, $d_k \geq 0$ is the random time delay from the controller to the actuator, and F_k is the controller at time instant k to be designed by the MPC method. Then control input $u(k)$ to the plant is:

$$u(k) = v(k - d_k) = F_{k-d_k} x(k - d_k - \tau_{k-d_k}) \tag{2.12}$$

By substituting equation 2.12 in equation 2.10, we get

$$x(k+1) = Ax(k) + Bu(k)F_{k-d_k} x(k - d_k - \tau_{k-d_k}) \tag{2.13}$$

In their analysis it was assumed that the states are directly available from the plant. But in practicality, the states are not readily available from the plant to be used by the controller. They are to be estimated.

So, the investigation carried out in [70] was without the consideration of observer or the state estimator.

The work presented in the following chapter **4** is an extension of the analysis carried out in [70] by taking into consideration an observer to estimate the states and the addition of buffer in the actuation channel to store the data received from the plant and by considering various other additional factors. Such type of considerations can further improve the analysis in terms of system performance versus the packet loss.

2.3 Conclusions

In this chapter, we have presented a survey of the main results in terms of different ways to model a network delay. The survey has outlined basic assumptions and has taken into considerations several technical views on the analysis and design procedures leading to stability and controller design of networked systems under consideration. The key emphasis here was on NCSs modeled using Markov chains with and without transition matrices. The work done by various author's in the area of network predictive control was also outlined in this chapter.

Chapter 3

MJLS WITH PACKETS DROPOUTS AND PARTIALLY KNOWN TRANSITION MATRIX

3.1 Introduction

Networked control systems (NCSs) , where the spatially distributed system components are connected through a communication network, are an attractive branch in control systems that has been receiving significant research interest in recent years. Their main features provide several advantages such as lack of wiring modularity, quick and easy maintenance and low cost, most of which have been very difficult to achieve in traditional point-to-point architecture. However, due to this, in networked control system, the packets may get dropped or delayed due to the network unreliability. The communication channel state does vary with time and has strong dependency on its error in the previous time instant. The best possible way to describe this data dropout process and capture the possible temporal correlation of network conditions is through Markov process. Rigorous research has been carried out in this domain to ensure better efficiency and stability of Networked Control Systems. The related literature are found in [90]-[30], where the effects of such data dropouts on the stability and performance has been addressed. In [57], the binary Markov process was adopted to model the packet loss and the criteria of covariance stability was treated. The results reported in [18, 12, 9] have thoroughly investigated the networked control systems where the delays in the channels are modeled Using Markov Chain. In these works, a plant was considered where the states of the network are modeled by a Markov chain and Lyapunov equations for the expected LQG performance was presented. An illustrative examples were also presented which supports the proposed theory. Some works have been published in recent years [44, 34, 20] were NCSs with packet dropouts on both sensor to controller (C/A) and controller to actuator (C/A) sides are considered. In [27], the authors discussed about the linear/nonlinear LQG optimal to minimize a cost function according to the transmission control protocol (TCP) and user datagram protocol (UDP). In their work, Markov chains were introduced to describe the S/C and C/A packet dropouts. The Markov chains were considered to describe the quantity of packet dropouts between current time k and its latest successful transmission instead of only the information on if a packet is dropped or not, which is different from the aforementioned references. By this definition, the number of states of Markov chains is larger than two and the history of packet dropouts can be seen clearly. In [13] quite similar results and analysis are presented, where the authors modeled a digital control systems with random but bounded delays in the feedback loop as finite-dimensional, discrete-time jump linear systems. The associated transition jumps were modeled as finite-state Markov chains. This class of systems can be called a stochastic hybrid system. It should bw noticed that all of the stability conditions and controller designs given in the aforementioned references are derived based on the assumption that transition probability matrices are known in advance.

Later on in [66], the stability analysis and stabilization problems for a class of discrete-time Markov jump linear systems were investigated with partially known transition probabilities and time-varying delays. The time-delay has a lower and upper bounds. In [89] a class of extended continuous-time Markov jump linear systems in was studied. Discrete dynamics of the class of systems is described by a Markov stochastic process, but with only partially known transition probabilities. In both of theise works, it was assumed that packet dropout existed only in the sensor-to-controller (S/C) side.

On another research front, systems with Markovian jump parameters are a class of hybrid systems which combine one part of the state taking values continuously and the other part of the state taking values discretely [37]. This class of systems are very popular in modeling practical systems where sudden environment changes, random failures and repairs may occur [38, 50, 54, 25, 19, 41]. Over the past decades, many researchers have devoted their efforts to the study of Markovian jump systems, and a great number of results have been available on the problems of stability analysis, controller design and state estimation, see, [33, 36, 67, 45, 68, 74] and the references therein.

In view of the foregoing results, we extend in this paper the work of [27]–[66] further and focus on the stability analysis and stabilization synthesis problems for a class of discrete-time Markov jump linear systems (MJLS) with partially known transition probabilities. Thus, sensor to controller (C/A) and controller to actuator (C/A) packet dropouts are described by Markov chains. These chains describe the quantity of packet dropouts between current time k and its latest successful transmission in addition to the information whether a packet is dropped or not. This defines the number of states of Markov chain to be larger than two and the packet dropouts history can be seen clearly. Through augmentation of the state vector, the result of the closed loop system can be transformed to a standard jump linear system with time delays. This enables us to implement the results of jump linear systems for the analysis and synthesis of such NCSs.

Specifically, our work mainly deals with the Markovian jump linear systems, where the controller gain is decided based on the number of packets dropped out on the sensor controller side. The main contribution of this paper is to take

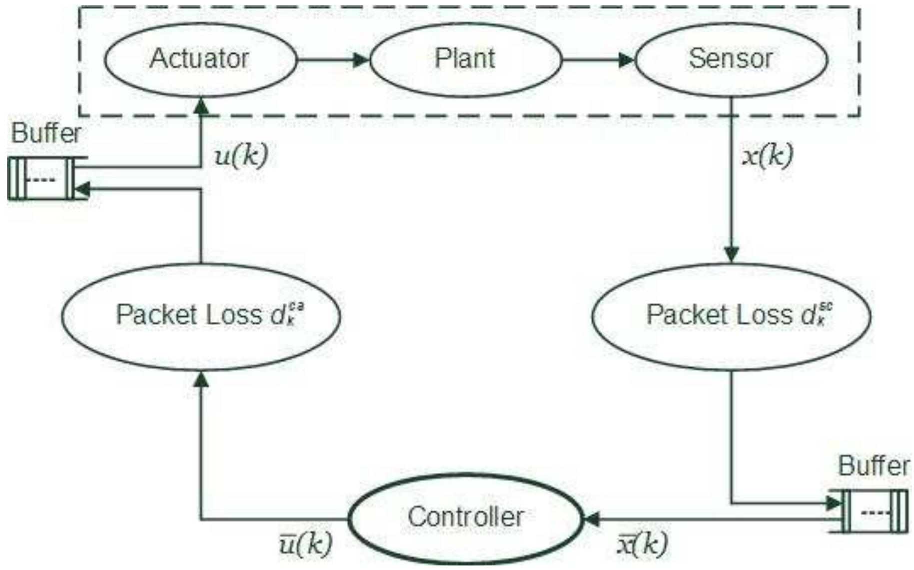

Figure 3.1: Networked control system with data dropout

in consideration the dropouts of the packets on both sensor to controller(S/A) side and controller and actuator(C/A) side. The transition probability matrices describing these packets dropouts on both the side S/C and C/A respectively are not known all the time. In fact, most of the time they are unknown and difficult to determine. In order to deal which such kind of practical situations where the transition matrices are not known completely, the author has proposed the technique through which the systems with unknown transition matrices can be controlled and made stable. According to author's knowledge so far such an issue has not been completely addressed in the literature [102, 27, 30].

The remaining part of the paper is organised as follows:Section II mainly deals with the problem description and formulation which is helpful throughout this paper and gives the basic idea of our setup. In section III, firstly, the stability analysis and controller design is discussed for completely known transition probabilities case and then the partially known case is considered. In Section IV, NCSs under multi-packets transmission is considered and the stability analysis and controller design is discussed as in single packet transmission case. Then in section V an illustrative example is given to demonstrate the effectiveness of the proposed theory. Finally in section VI we conclude the work.

Notations: The notations given in this paper are fairly standard.The super script $''T''$ stands for matrix transposition, $\mathbb{R}^n$ denotes the n dimensional Euclidean space. The $diag\{\cdots\}$ represents the block diagonal matrix. I and 0 stands for identity matrix and zero matrix respectively. In symmetric block matrices or complex matrix expressions, we use the symbol $\bullet$ to represent a term that is induced by symmetry. Sometimes, the arguments of a function will be omitted when no confusion can arise.

3.2 Problem Formulation

A networked controlled system is considered as shown in fig. 3.1. This NCS setup is assumed to suffer from the transmission delay possibly due to the packets dropout at S/C and C/A side. Here the sensors, controllers, and actuators are all clock-driven. The linear time-invariant (LTI) plant we consider here is [89]

$$x(k+1) = \phi x(k) + \Gamma u(k) \tag{3.1}$$

where $x(k) \in \Re^n$ is the plant state vector and $u(k) \in \Re^m$ and ϕ and Γ are known real constant matrices with appropriate dimensions. It is supposed that the buffers are long enough to hold all the arrived packets, which will be picked up using the last-in-first-out rule. For example, during the transmission from sensor to controller, the measured state $x(k)$ is lost, the previous value $x(k-1)$ will be read out by the controller from the buffer which is utilized as $\bar{x}(k)$ for calculating the new control signal, which is supplied to the plant; Else, the current value of $x(k)$ will be used by the controller as $\bar{x}(k)$ and will also be saved in the buffer. Thus, for the buffers, we have

$$u(k) = \begin{cases} \bar{u}(k) & \text{if successfully transmitted} \\ u(k-1) & \text{otherwise} \end{cases} \tag{3.2}$$

$$\bar{x}(k) = \begin{cases} x(k) & \text{if successfully transmitted} \\ \bar{x}(k-1) & \text{otherwise} \end{cases} \tag{3.3}$$

The number of packets dropped at time k in the measurement channel is assumed to be d_k^{sc}, which is computed from the current time k to the last successful transmission (happened at time $k - d_k^{sc}$), d_k^{ca} is the packet quantity dropped on the C/A side between the current time k and its last successful transmission at time $k - d_k^{ca}$, and they are bounded. Thus, we have

$$0 \leq d_k^{sc} \leq d_1, \;\; 0 \leq d_k^{ca} \leq d_2$$

where d_1 and d_2 are real positive integers. d_k^{sc} and d_k^{ca} are two independent homogeneous Markov chains, which takes values in $S_1 = \{0, 1, \cdots, d_1\}$ and $S_2 = \{0, 1, \cdots, d_2\}$. The transition probabilities matrices are defined by

$$\rho_{ij} = Pr(d_{k+1}^{sc} = j | d_k^{sc} = i), \;\; \lambda_{mn} = Pr(d_{k+1}^{ca} = n | d_k^{ca} = m) \tag{3.4}$$

Hence

$$\rho = \begin{bmatrix} \rho_{11} & \rho_{12} \cdots \rho_{1N} \\ \rho_{21} & \rho_{22} \cdots \rho_{2N} \\ & \\ \rho_{N1} & \rho_{N2} \cdots \rho_{NN} \end{bmatrix}, \quad \lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \cdots \lambda_{1N} \\ \lambda_{21} & \lambda_{22} \cdots \lambda_{2N} \\ & \\ \lambda_{N1} & \lambda_{N2} \cdots \lambda_{NN} \end{bmatrix}$$

In addition, it is considered that the transition probabilities of the Markov chain are partially available, as, some of the elements in matrix ρ and λ do not vary with time and are not known.

For instance, a system (3.1) with three modes will have the transition probabilities matrices, ρ and λ as

$$\rho = \begin{bmatrix} \rho_{11} & \rho_{12} & ? \\ ? & \rho_{22} & \rho_{32} \\ \rho_{31} & ? & ? \end{bmatrix}, \quad \lambda = \begin{bmatrix} ? & \lambda_{12} & ? \\ ? & ? & \lambda_{32} \\ ? & ? & ? \end{bmatrix}$$

where the unknown elements are represented by "?". For notation clarity, $\forall i \in \mathcal{I}$, we denote

$\mathcal{I}^i_{\mathcal{K}} \triangleq \{j$:if ρ_{ij} is known$\}$, $\mathcal{I}^i_{u\mathcal{K}} \triangleq \{j$:if ρ_{ij} is unknown $\}$

$\mathcal{I}^m_{\mathcal{K}} \triangleq \{n$:if λ_{mn} is known $\}$, $\mathcal{I}^m_{u\mathcal{K}} \triangleq \{n$:if λ_{mn} is unknown$\}$

Consider the state feedback control law as

$$\bar{u}(k) = F(d_k^{sc})\bar{x}(k) \tag{3.5}$$

where $F(d_k^{sc})$ is a controller-set designed based on d_k^{sc}. Substituting (3.2) and $\bar{u}(k)$ into the system in (3.1), we get the following closed-loop system:

$$x(k+1) = \begin{cases} \phi x(k) + \Gamma F(d_k^{sc})\bar{x}(k) & \text{if } d_k^{ca} = 0 \\ \\ \phi x(k) + \Gamma u(k-1) & \text{otherwise } d_k^{ca} > 0 \end{cases} \tag{3.6}$$

Note that $\bar{x}(k) = x(k - d_k^{sc})$. In order to simplify the equation of the closed-loop system, a function $\alpha(\cdot)$ is introduced to augment the previous closed-loop system as

$$\begin{aligned} x(k+1) &= \phi x(k) + \alpha(d_k^{ca})\Gamma u(k-1) \\ &+ [1 - \alpha(d_k^{ca})]\Gamma F(d_k^{sc})x(k - d_k^{sc}) \end{aligned} \tag{3.7}$$

$$\begin{aligned} u(k) &= \alpha(d_k^{ca})u(k-1) \\ &+ [1 - \alpha(d_k^{ca})]F(d_k^{sc})x(k - d_k^{sc}) \end{aligned} \tag{3.8}$$

where

$$\alpha(d_k^{ca}) = \begin{cases} 1 & d_k^{ca} > 0 \\ \\ 0 & d_k^{ca} = 0 \end{cases}$$

Remark 3.2.1 *: The value of $\alpha(.)$ depends on d_k^{ca}, which indicates whether a successful transmission of the control signal to the actuator has taken place or not (namely, $d_k^{ca} = 0$ or $d_k^{ca} > 0$), rather than just giving the information about the quantity of control signals which are dropped(the value of d_k^{ca}). The closed loop system modeling is simplified by the introduction of $\alpha(\cdot)$, as the control input $u(k)$ will no longer be updated with regards to any value of $d_k^{ca} > 0$. This implies that, the value of control signal $u(k)$ will be same for any $d_k^{ca} = 1, 2, 3, \cdots d_2$. Another advantage obtained through this classification that the unknown d_k^{ca} is avoided from being introduced directly in the state vectors of the augmented system. Thus, we replace $u(k)$ with (3.2) instead of $u(k) = \bar{u}(k - d_k^{ca})$, the method of computation being similar to the recursive method for $\bar{x}(k)$.*

By the Concatenation of plant and controller state vectors we obtain a global vector $z(k) = [x^T(k)\ u^T(k-1)]$. Thus the closed-loop system obtained for the NCS with single-packet transmissions is

$$\begin{aligned} z(k+1) &= \begin{bmatrix} \phi & \Gamma\alpha(d_k^{ca}) \\ 0 & \alpha(d_k^{ca}) \end{bmatrix} \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix} \\ &+ \begin{bmatrix} (1-\alpha(d_k^{ca}))\Gamma F(d_k^{sc}) & 0 \\ (1-\alpha(d_k^{ca}))F(d_k^{sc}) & 0 \end{bmatrix} \begin{bmatrix} x(k-d_k^{sc}) \\ u(k-d_k^{sc}-1) \end{bmatrix} \end{aligned}$$

$$z(k+1) = A(d_k^{ca})z(k) + B(d_k^{sc}, d_k^{ca})z(k-d_k^{sc}) \tag{3.9}$$

Therefore the obtained closed-loop system is identified as a Markovian jump linear system(MJLs) having two modes (d_k^{sc} and d_k^{ca}) and one time-varying mode-dependent delay d_k^{sc}. This also allow us to implement the results of MJLs with time-delays for the analysis and synthesis of such NCSs.

For an accurate description of the main objective of this paper, the following definition is introduced for the system under consideration.

Definition 1: System (3.9) is said to be stochastically stable if for all finite $z(k) = \phi \in \Re^{\bar{n}+\bar{m}}$ defined on $k \in [-d_1, 0]$ and initial model d_0^{sc}, d_0^{ca}, there exists a finite number $\mathbf{E}(\phi, d_0^{sc}, d_0^{ca}) > 0$ such that,

$$\lim_{N\to+\infty} \mathbf{E}\{\sum_{k=0}^{N} \| z(k) \|^2 \mid \phi(.), d_0^{sc}, d_0^{ca}\} < \mathbf{E}(\phi, d_0^{sc}, d_0^{sc}) \tag{3.10}$$

holds, where $\mathbf{E}$ is the statistical expectation operator.

3.3 NCSs with Completely and Partially known Transition Matrices

In following section, the sufficient conditions required for the stochastic stability of the closed loop system with completely known transition probability matrices is derived first in **Theorem 1**. Later the case of partially known transition matrices is considered and corresponding controller design is given in **Theorem 2**. For notational simplicity, in the sequel, for $d_k^{sc} = i \in S_1, d_k^{ca} = m \in S_2$, we denote $A(d_k^{ca}) \triangleq A_m,\ B(d_k^{sc}, d_k^{ca}) \triangleq B(i,m)$ and let $k = (I + \Gamma^T\Gamma)^{-1}[\Gamma^T \ \ I], \underline{d_1} = min\{d_k^{sc}, k \in \mathbb{Z} i \in S_1\}, \underline{\rho} = min\{\rho_{ii}, i \in S_1\}$. Then, we have

Theorem 1: Consider system given in (3.9) with completely known transition probability matrices. The system is said be stochastically stable if there exist matrices $P(i,m) > 0, B(i,m) > 0, Q > 0, R > 0, X_v > 0, v = 1, 2, M_{(i,m)(v)}$,

$N_{(i,m)(v)}, S_{(i,m)(v)}, v = 1, 2, 3, \forall i, m \in \mathcal{I}$ such that

$$\begin{bmatrix} -\bar{P}(i,m) & 0 & 0 & \omega_{(i,m)1} \\ \bullet & -X_2 & 0 & \omega_{(i,m)2} \\ \bullet & \bullet & -X_1 & \omega_{(i,m)3} \\ \bullet & \bullet & \bullet & \omega_{(i,m)4} \end{bmatrix} < 0 \tag{3.11}$$

where

$$\begin{aligned}
\omega_{(i,m)1} &\triangleq \begin{bmatrix} \bar{P}(i,m)A_m & \bar{P}_{i,m}B(i,m) & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\omega_{(i,m)2} &\triangleq \begin{bmatrix} \sqrt{d_1}X_2(A_m - I) & \sqrt{d_1}X_2B(i,m) & 0 & 0 & 0 & 0 \end{bmatrix} \\
\omega_{(i,m)3} &\triangleq \begin{bmatrix} \sqrt{d_1}X_1(A_m - I) & \sqrt{d_1}X_1B(i,m) & 0 & 0 & 0 & 0 \end{bmatrix} \\
\omega_{(i,m)4} &\triangleq \begin{bmatrix} \eta_{(i,m)11} & \eta_{(i,m)12} & \eta_{(i,m)13} & \sqrt{d_1}M_{(i,m)1} & \sqrt{d_1 - \underline{d}_1}S_{(i,m)1} & \sqrt{d_1}N_{(i,m)1} \\ \bullet & \eta_{(i,m)22} & \eta_{(i,m)23} & \sqrt{d_1}M_{(i,m)2} & \sqrt{d_1 - \underline{d}_1}S_{(i,m)2} & \sqrt{d_1}N_{(i,m)2} \\ \bullet & \bullet & \eta_{(i,m)33} & \sqrt{d_1}M_{(i,m)3} & \sqrt{d_1 - \underline{d}_1}S_{(i,m)3} & \sqrt{d_1}N_{(i,m)3} \\ \bullet & \bullet & \bullet & -X_1 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -X_1 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -X_2 \end{bmatrix}
\end{aligned}$$

$\bar{P}_{i,m} \triangleq \sum_{j\in\mathcal{I}} \sum_{n\in\mathcal{I}} \rho_{ij}\lambda_{mn}P(j,n)$

$$
\begin{aligned}
\eta_{(i,m)11} &\triangleq -P(i,m) + (1 + d_1 - \underline{d}_1)Q + R + M_{(i,m)1} \\
&\quad + N_{(i,m)1} + M_{(i,m)1}^T + N_{(i,m)1}^T \\
\eta_{(i,m)12} &\triangleq S_{(i,m)1} - M_{(i,m)1} + M_{(i,m)2}^T + N_{(i,m)2}^T \\
\eta_{(i,m)13} &\triangleq -N_{(i,m)1} - S_{(i,m)1} + M_{(i,m)3}^T + N_{(i,m)3}^T \\
\eta_{(i,m)22} &\triangleq -Q + S_{(i,m)2} - M_{(i,m)2} + S_{(i,m)2}^T - M_{(i,m)2}^T \\
\eta_{(i,m)23} &\triangleq -N_{(i,m)2} - S_{(i,m)2} + S_{(i,m)3}^T - M_{(i,m)3}^T \\
\eta_{(i,m)33} &\triangleq -R - N_{(i,m)3} - S_{(i,m)3} - N_{(i,m)3}^T - S_{(i,m)3}^T
\end{aligned}
$$

Proof: Let the Lyapunov-Krasovskii functional be

$$
\begin{aligned}
V(z(k),k) &= \sum_{s=1}^{5} V_s(z(k),k) \\
V_1(z(k),k) &= z^T(k)P(i,m)z(k) \\
V_2(z(k),k) &= \sum_{\tau=k-d_k^{sc}}^{k-1} z^T(\tau)Qz(\tau) \\
V_3(z(k),k) &= \sum_{\tau=k-d_1}^{k-1} z^T(\tau)Rz(\tau) \\
V_4(z(k),k) &= (1-\underline{\rho}) \sum_{\theta=-d_1+1}^{-\underline{d}_1} \sum_{\tau=k+\theta}^{k-1} z^T(\tau)Qz(\tau) \\
V_5(z(k),k) &= \sum_{\theta=-d_1}^{-1} \sum_{\tau=k+\theta}^{k-1} y^T(\tau)(X_1+X_2)y(\tau)
\end{aligned}
$$

with $y(\tau) \triangleq z(\tau+1)-z(\tau)$ and $P(i,m), B(i,m), Q, R, X_1, X_2, M_{(i,m)v}, N_{(i,m)v}, S_{(i,m)v}$ are to be determined. Thus we have,

$$\begin{aligned}
\Delta V_1 &\triangleq \mathbf{E}[V_1(z(k+1,k+1|z(k),k)) - V_1(z(k),k)] \\
&= z^T(k+1)\sum_{n=0}^{d_2}\sum_{j=0}^{d_1}\lambda_{mn}\rho_{ij}P(j,n)z(k+1) \\
&\quad -z^T(k)P(i,m)z(k) \\
&= z^T(k)(A_m^T\bar{P}(i,m)A_m - P(i,m))z(k) \\
&\quad +2z^T(k)A_m^T\bar{P}(i,m)B(i,m)z(k-d_k^{sc}) \\
&\quad +z^T(k-d_k^{sc})B^T(i,m)\bar{P}(i,m)B(i,m)z(k-d_k^{sc})
\end{aligned}$$

$$\begin{aligned}
\Delta V_2 &\triangleq \mathbf{E}[V_2(z(k+1,k+1|z(k),k)) - V_2(z(k),k)] \\
&= (\sum_{\tau=k+1-d_{k+1}^{sc}}^{k} - \sum_{\tau=k-d_k^{sc}}^{k-1})z^T(\tau)Qz(\tau) \\
&= z^T(k)Qz(k) - z^T(k-d_k^{sc})Qz(k-d_k^{sc}) \\
&\quad + \sum_{\tau=k+1-d_{k+1}^{sc}}^{k-d_{k+1}^{sc}} z^T(\tau)Qz^(\tau) \\
&\le z^T(k)Qz(k) - z^T(k-d_k^{sc})Qz(k-d_k^{sc}) \\
&\quad + \sum_{\tau=k-d_1+1}^{k-\underline{d_1}} z^T(\tau)Qz(\tau) \\
\Delta V_3 &\triangleq \mathbf{E}[V_3(z(k+1,k+1|z(k),k)) - V_3(z(k),k)]
\end{aligned}$$

$$
\begin{aligned}
&= (\sum_{\tau=k+1-d_1}^{k} - \sum_{\tau=k-d_1}^{k-1})z^T(\tau)Rz(\tau) \\
&= z^T(k)Rz(k) - z^T(k-d_1)Rz(k-d_1) \\
\Delta V_4 &\triangleq \mathbf{E}[V_4(z(k+1,k+1|z(k),k)) - V_4(z(k),k)] \\
&= \sum_{\theta=-d_1+1}^{-\underline{d_1}} (\sum_{\tau=k+1+\theta}^{k} - \sum_{\tau=k+\theta}^{k-1})z^T(\tau)Rz(\tau) \\
&= (d_1 - \underline{d_1})z^T(\tau)Rz(\tau) \\
&- (1-\underline{\rho}) \sum_{\tau=k-d_1+1}^{k-\underline{d_1}} z^T(\tau)Rz(\tau) \\
\Delta V_5 &\triangleq \sum_{\theta=-d_1}^{-1} [\sum_{\tau=k+\theta+1}^{k} y^T(\tau)(X_1+X_2)y(\tau) \\
&- \sum_{\tau=k+\theta}^{k-1} y^T(\tau)(X_1+X_2)y(\tau)] \\
&= \sum_{\theta=-d_1}^{-1} (\sum_{\tau=k+\theta+1}^{k} - \sum_{\theta=k+\theta}^{k-1})y^T(\tau)(X_1+X_2)y(\tau) \\
&= \sum_{\theta=-d_1}^{-1} [y^T(k)(X_1+X_2)y(k) \\
&- y^T(k+\theta)(X_1+X_2)y(k+\theta)] \\
&= d_1 y^T(k)(X_1+X_2)y(k) - \sum_{\tau=k-d_k^{sc}}^{k-1} y^T(\tau)X_1y(\tau) \\
&- \sum_{\tau=k-d_1}^{k-d_k^{sc}-1} y^T(\tau)X_1y(\tau) - \sum_{\tau=k-d_1}^{k-1} y^T(\tau)X_2y(\tau)
\end{aligned}
$$

then we have

$$\begin{aligned}
\Delta V(z(k),k) &= z^T(k)(A_m^T\bar{P}(i,m)-P(i,m))z(k)\\
&+ 2z^T(k)A_m^T\bar{P}(i,m)B^T(i,m)z(k-d_k^{sc})\\
&+ z^T(k-d_k^{sc})B^T(i,m)\bar{P}(i,m)B(i,m)z(k-d_k^{sc})\\
&- z^T(k-d_k^{sc})Rz(k-d_k^{sc})\\
&+ (d_1-\underline{d_1}+1)z^T(k)Rz(k)\\
&+ z^T(k)Rz(k)-z^T(k-d_1)Rz(k-d_1)\\
&+ d_1[(A_m-I)z(k)+B(i,m)z(k-d_k^{sc})]^T\\
&\times (X_1+X_2)[(A_i-I)z(k)+B(i,m)z(k-d_k^{sc})]\\
&- \sum_{\tau=k-d_k^{sc}}^{k-1} y^T(\tau)X_1y(\tau)-\sum_{\tau=k-d_1}^{k-d_k^{sc}-1} y^T(\tau)X_1y(\tau)\\
&- \sum_{\tau=k-d_1}^{k-1} y^T(\tau)X_2y(\tau)\\
&+ 2\zeta^T(k)M_{(i,m)}[z(k)-z(k-d_k^{sc})-\sum_{\tau=k-d_k^{sc}}^{k-1} y(\tau)]\\
&+ 2\zeta^T(k)S_{(i,m)}[z(k-d_k^{sc})-z(k-d_1)-\sum_{\tau=k-d_1}^{k-d_k^{sc}-1} y(\tau)]\\
&+ 2\zeta^T(k)N_{(i,m)}[z(k)-z(k-d_1)-\sum_{\tau=k-d_1}^{k-1} y(\tau)]
\end{aligned}$$

Therefore, we obtain

$$
\begin{aligned}
\Delta V(z(k),k) &\leq \zeta^T(k)[\Omega_{(i,m)} + \Theta_{(i,m)} + d_1 M_{(i,m)} X_1^{-1} M_{(i,m)}^T + (d_1 - \underline{d_1}) \times S_{(i,m)} X_1^{-1} S_{(i,m)}^T \\
&+ d_1 N_{(i,m)} X_2^{-1} N_{(i,m)}^{-1}]\zeta(k) \\
&- \sum_{\tau=k-d_k^{sc}}^{k-1} [\zeta^T(k) M_{(i,m)} + y^T(\tau) X_1] X_1^{-1} [\zeta^T(k) M_{(i,m)} + y^T(\tau) X_1]^T \\
&- \sum_{\tau=k-d_1}^{k-d_k^{sc}-1} [\zeta^T(k) S_{(i,m)} + y^T(\tau) X_1] X_1^{-1} [\zeta^T(k) S_{(i,m)} + y^T(\tau) X_1]^T \\
&- \sum_{\tau=k-d_1}^{k-1} [\zeta^T(k) N_{(i,m)} + y^T(\tau) X_2] X_2^{-1} [\zeta^T(k) N_{(i,m)} + y^T(\tau) X_2]^T
\end{aligned}
$$

where

$$
\begin{aligned}
\zeta &\triangleq \begin{bmatrix} z(k)^T & z^T(k-d_k^{sc}) & z^T(k-d_1) \end{bmatrix}, \\
\Omega_{(i,m)} &\triangleq \begin{bmatrix} \Omega_{(i,m)1} & \Omega_{(i,m)2} & 0 \\ \bullet & \Omega_{(i,m)3} & 0 \\ \bullet & \bullet & -R \end{bmatrix}, \\
\Theta_{(i,m)} &\triangleq \begin{bmatrix} M_{(i,m)} + N_{(i,m)} & S_{(i,m)} - M_{(i,m)} & -S_{(i,m)} - N_{(i,m)} \end{bmatrix}, \\
&+ \begin{bmatrix} M_{(i,m)} + N_{(i,m)} & S_{(i,m)} - M_{(i,m)} & -S_{(i,m)} - N_{(i,m)} \end{bmatrix}^T
\end{aligned}
$$

Since both $X_1 > 0$ and $X_2 > 0$, the last three terms are non positive in $\Delta V(z(k),k)$. By Schur complement, (3.11) guarantees $[\Omega_{(i,m)} + \Theta_{(i,m)} + d_1 M_{(i,m)} X_1^{-1} M_{(i,m)}^T + (d_1 - \underline{d_1}) \times S_{(i,m)} X_1^{-1} S_{(i,m)}^T + d_1 N_{(i,m)} X_2^{-1} N_{(i,m)}^{-1}] < 0$. Therefore, we have $\Delta V(z(k),k) <$

$-\delta\|z(k)\|^2$ for a sufficiently small $\delta > 0$ and $z(k) \neq 0$.

Theorem 2: Let us consider the system (3.9) having partially known transition probability matrices. There exists a controller (3.5) such that the resulting closed-loop system is stochastically stable if there exist matrices $P(i,m) > 0, i, m \in I, B(i,m) > 0, Q > 0, R > 0, X_v > 0, v = 1, 2, M_{(i,m)}, N_{(i,m)}, S_{(i,m)}, v = 1, 2, 3, \forall i, m \in \mathcal{I}$ such that

$$\begin{bmatrix} -\chi_{j,n} & 0 & 0 & \omega_{(i,m)5} \\ \bullet & -X_2 & 0 & \omega_{(i,m)2} \\ \bullet & \bullet & -X_1 & \omega_{(i,m)3} \\ \bullet & \bullet & \bullet & \omega_{(i,m)4} \end{bmatrix} < 0 \tag{3.12}$$

where

$$\omega_{(i,m)5} \triangleq \begin{bmatrix} \chi(j,n)A_m & \chi(j,n)B(i,m) & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$\omega_{(i,m)v}, v = 2, 3, 4$ are as defined in *Theorem 1*

if $\rho^i_{\mathcal{K}} = 0$ and $\lambda^m_{\mathcal{K}} = 0$, then $\chi(j,n) \triangleq P(j,n)$, otherwise

$$\chi(j,n) = \begin{cases} \frac{1}{\lambda^m_{\mathcal{K}}\rho^i_{\mathcal{K}}} P^{(i,m)}_{\mathcal{K}} \\ P(j,n), \forall j \in \mathcal{I}^i_{\mathcal{UK}}, \forall n \in \mathcal{I}^m_{\mathcal{UK}} \end{cases}$$

with $P_{\mathcal{K}}^{(i,m)} \triangleq \sum_{j\in\mathcal{I}_{\mathcal{K}}^{i}} \sum_{n\in\mathcal{I}_{\mathcal{K}}^{m}} \lambda_{mn}\rho_{ij}P(j,n)$

Proof: First, we know that the system (3.9) is stochastically stable under the completely known transition probabilities if (3.11) satisfies. Note that (3.11) can be rewritten as

$$\begin{aligned}
\mathbf{E}_{(i,m)} &\triangleq \begin{bmatrix} -P_{\mathcal{K}}^{(i,m)} & P_{\mathcal{K}}^{(i,m)}\mathbf{E}_{(i,m)1} \\ \\ \bullet & \lambda_{\mathcal{K}}^{m}\rho_{\mathcal{K}}^{i}\mathbf{E}_{(i,m)2} \end{bmatrix} \\
&\quad + \sum_{j\in\mathcal{I}_{\mathcal{UK}}^{i}} \sum_{n\in\mathcal{I}_{\mathcal{UK}}^{m}} \lambda_{mn}\rho_{ij} \begin{bmatrix} -P(j,n) & P(j,n)\mathbf{E}_{(i,m)1} \\ \bullet & \mathbf{E}_{(i,m)2} \end{bmatrix}
\end{aligned}$$

where

$$\begin{aligned}
\mathbf{E}_{(i,m)1} &\triangleq \begin{bmatrix} 0 & 0 & A_m & B(i,m) & 0 & 0 & 0 & 0 \end{bmatrix} \\
\mathbf{E}_{(i,m)2} &\triangleq \begin{bmatrix} -X_2 & 0 & \omega_{(i,m)2} \\ \bullet & -Z_1 & \omega_{(i,m)3} \\ \bullet & \bullet & \omega_{(i,m)4} \end{bmatrix}
\end{aligned}$$

Therefore if one has

$$\begin{bmatrix} -P_{\mathcal{K}}^{(i,m)} & P_{\mathcal{K}}^{(i,m)}\mathbf{E}_{(i,m)1} \\ \\ \bullet & \lambda_{\mathcal{K}}^{m}\rho_{\mathcal{K}}^{i}\mathbf{E}_{(i,m)2} \end{bmatrix} < 0 \tag{3.13}$$

$$\begin{bmatrix} -P(j,n) & P(j,n)\mathbf{E}_{(i,m)1} \\ \bullet & \mathbf{E}_{(i,m)2} \end{bmatrix} < 0, \tag{3.14}$$
$$\forall j \in \mathcal{I}_{\mathcal{UK}}^{i}, \forall n \in \mathcal{I}_{\mathcal{UK}}^{m}$$

then we have $\mathbf{E}_{(i,m)} < 0$, hence the system is stochastically stable under partially known transition probabilities, which is concluded from the obvious fact that no knowledge on $\rho_{ij} \forall j \in \mathcal{I}_{\mathcal{UK}}^{i}$ and $\lambda_{mn} \forall n \in \mathcal{I}_{\mathcal{UK}}^{m}$, is required in (3.13) and (3.14). Thus, for $\rho_{\mathcal{K}}^{i}, \lambda_{\mathcal{K}}^{m} \neq 0$ and $\rho_{\mathcal{K}}^{i}, \lambda_{\mathcal{K}}^{m} = 0$,, respectively, one can readily obtain (3.12), since if $\rho_{\mathcal{K}}^{i}, \lambda_{\mathcal{K}}^{m} = 0$ the conditions (3.13), (3.14) will reduce to (3.14). This completes the proof.

To determine the controller F_i given in (3.5), consider that $\psi_{(i,m)} = X_1 B^T(i,m)$, where $X_2 = X_1^{-1}$.Therefore $\psi_{(i,m)}$ is given as

$$\psi_{(i,m)} = (1-\alpha(d_k^{ca}))X_2 \begin{bmatrix} \Gamma F_i & 0 \\ F_i & 0 \end{bmatrix}^T$$
$$= (1-\alpha(d_k^{ca}))X_2 \begin{bmatrix} I \\ 0 \end{bmatrix} F_i^T \begin{bmatrix} \Gamma^T & I \end{bmatrix}$$

The controller F_i is obtained by premultiplying $\begin{bmatrix} I & 0 \end{bmatrix} X_2^{-1}$ and postmultiplying $\begin{bmatrix} \Gamma^T & I \end{bmatrix}^T$ to both sides of above equation.Note that $\Gamma^T\Gamma + I$ is of full rank.

Hence, the controller equation is obtained as

$$u(k) = \begin{cases} \bar{u}(k-1) & \text{if} \quad d_k^{ca} > 0 \\ K\psi_{(i,0)}^T \begin{bmatrix} I & 0 \end{bmatrix}^T X_2^{-1} x(k - d_k^{sc}) & \text{if} \quad d_k^{ca} = 0 \end{cases}$$

where

$$K = (I + \Gamma^T\Gamma)^{-1} \begin{bmatrix} \Gamma^T & I \end{bmatrix}^T.$$

$$\psi_{(i,m)} = \begin{cases} Q \begin{bmatrix} I \\ 0 \end{bmatrix} F_i^T \begin{bmatrix} \Gamma^T & I \end{bmatrix} & \text{if} \quad d_k^{ca} = m = 0 \\ 0 & \text{if} \quad d_k^{ca} = m > 0 \end{cases}$$

3.4 NCSs under Multiple-Packet Transmissions

Single-packet transmission refers to the lumping of sensor or actuator data into one network packet, whereas in multiple-packet transmission, separate network packets are used for sensor or actuator data transmission.

A great deal of research has gone into the study of effect of packet dropouts on the stability of the NCS considering multiple-packet transmission policies. A similar study was carried out in [23], where the authors have considered dual packet transmissions. The optimal control problem for an NCS with the consideration of general multiple-packet transmission policy has been discussed in [35]. The control law was determined by the minimization of the cost function and was deployed for the stability analysis of the close-loop system. It was examined that if the norm of a certain matrix recursively defined converges to a number less than 1, a steady-state control policy exists. In [58] the authors gave stability analysis of an NCS under a multiple- packet transmission policy. This analysis was carried out for two cases: In case one, the communication channel is particularly assumed to be characterized by a known PDP, and case two only an upper bound on the probability is known.

However the stability condition and the controller design for the NCS under a multiple- packet transmission policy with partially known transition matrices was not completely addressed in the literature.

3.4.1 Modeling NCSs with multiple-packet transmissions

According to the characteristic mentioned in the previous section, the NCSs are modeled with $M + N(M + N > 2)$ Markov chains. The plant states are splitted into M separate packets as $x(k) = [X_1^T(k), X_2^T(k), \cdots, X_M^T(k)]^T$ and the controller output into N separate packets as $\bar{u}(k) = [\bar{U}_1^T(k), \bar{U}_2^T(k), \cdots, \bar{U}_N^T(k)]^T$, where

$$\begin{aligned} X_1(k) &= [x_1^T(k), x_2^T(k), \cdots, x_{G1}^T(k)]^T \\ \vdots \; &\; \vdots \\ X_M(k) &= [x_{G_{M-1}+1}^T(k), x_{G_{M-1}+2}^T(k), \cdots, x_{\bar{n}}^T(k)]^T \end{aligned}$$

and $1 \leq G_1 < \cdots < G_{M-1} \leq \bar{n}$; then, we have the corresponding input of the controller $\bar{x}(k)$ as

$$\begin{aligned}
\bar{x}(k) &= [\bar{X}_1^T(k), \bar{X}_2^T(k), \cdots, \bar{X}_M^T(k)]^T \\
\bar{X}_1(k) &= [\bar{x}_1^T(k), \bar{x}_2^T(k), \cdots, \bar{x}_{G1}^T(k)]^T \\
\vdots \; &\; \vdots \\
\bar{X}_M(k) &= [\bar{x}_{G_{M-1}+1}^T(k), \bar{x}_{G_{M-1}+2}^T(k), \cdots, \bar{x}_{\bar{n}}^T(k)]^T
\end{aligned}$$

Signals $\bar{u}(k)$ and $u(k)$ ave a similar definitions, and are omitted. In order to make our analysis simple, we assume two-packet transmission on the S/C side and single-packet transmission on the C/A side; i.e.,$M = 2$ and $N = 1$. Thus

$$\begin{aligned}
\bar{x}(k) &= \begin{bmatrix} \bar{X}_1(k) \\ \bar{X}_2(k) \end{bmatrix} = \begin{bmatrix} X_1(k - d_{1k}^{sc}) \\ X_2(k - d_{2k}^{sc}) \end{bmatrix} \\
u(k) &= U_1(k) = \bar{U}_1(k - d_k^{ca})
\end{aligned}$$

where, $G_1 = G; d_{1k}^{sc}$ and d_{2k}^{sc} indicates the S/C packet dropouts quantities in channels 1 and 2, and d_k^{ca} indicates the C/A packet dropouts quantity. Their transition probabilities are given by

$$
\begin{aligned}
&\rho_{ij} = Pr(d^{sc}_{1(k+1)} = j | d^{sc}_{1k} = i) \\
&\rho_{ij} \geq 0, \quad i,j \in S_{11}\{0,1,\cdots,d_{11}\} \\
&\pi_{rq} = Pr(d^{sc}_{2(k+1)} = q | d^{sc}_{2k} = r) \\
&\pi_{rq} \geq 0, \quad r,q \in S_{12}\{0,1,\cdots,d_{12}\} \\
&\lambda_{mn} = Pr(d^{ca}_{k+1} = n | d^{ca}_{k} = m) \\
&\lambda_{mn} \geq 0, \quad m,n \in S_{2}\{0,1,\cdots,d_{2}\} \qquad (3.15)
\end{aligned}
$$

Let the transition probability matrix π with three modes be modeled as,

$$
\pi = \begin{bmatrix} \pi_{11} & \pi_{12} & ? \\ ? & \pi_{22} & \pi_{32} \\ \pi_{31} & ? & ? \end{bmatrix}
$$

where "?" represents the unavailable elements. For notation clarity, $\forall r \in \mathcal{I}$, we denote

$\mathcal{I}^r_{\mathcal{K}} \triangleq \{q$:if π_{rq} is known$\}$, $\mathcal{I}^r_{u\mathcal{K}} \triangleq \{q$:if π_{rq} is unknown $\}$

As in the single-packet transmission case, the closed-loop system in this case can be modeled as

$$
\begin{aligned}
z(k+1) &= \begin{bmatrix} \phi & \Gamma\alpha(d_k^{ca}) \\ 0 & \alpha(d_k^{ca}) \end{bmatrix} z(k) \\
&+ \begin{bmatrix} \Gamma\Pi_1 & 0 \\ \Pi_1 & 0 \end{bmatrix} z(k-d_{1k}^{sc}) \\
&+ \begin{bmatrix} \Gamma\Pi_2 & 0 \\ \Pi_2 & 0 \end{bmatrix} z(k-d_{2k}^{sc})
\end{aligned}
$$

where

$$
\begin{aligned}
\Pi_1 &= (1-\alpha(d_k^{ca}))F_{\bar{m}\times G}(d_{1k}^{sc}) \begin{bmatrix} I_G & 0_{G\times(\bar{n}-G)} \end{bmatrix} \\
\Pi_2 &= (1-\alpha(d_k^{ca}))F_{\bar{m}\times(\bar{n}-G)}(d_{2k}^{sc}) \begin{bmatrix} 0_{(\bar{n}-G)\times G} & 0_{\bar{n}\times G} \end{bmatrix}
\end{aligned}
$$

The closed loop equation finally obtained is

$$
\begin{aligned}
z(k+1) &= A(d_k^{ca})z(k) + B_1(d_k^{ca}, d_{1k}^{sc})z(k-d_{1k}^{sc}) \\
&+ B_2(d_k^{ca}, d_{2k}^{sc})z(k-d_{2k}^{sc})
\end{aligned} \tag{3.16}
$$

Complexity of the closed-loop models depends on value of M and N:Larger the value of M and N, more complicated models becomes. However, the method used above to model the NCSs under multiple-packet transmission will not affect our stability analysis and controller design;the only complications are observed in mathematical calculations.

Definition 2: System (3.16) is said to be stochastically stable if for all finite $z(k) = \phi \in \Re^{\bar{n}+\bar{m}}$ defined on $k \in [-max(d_{11}, d_{12}), 0]$ and initial model $d_{10}^{sc}, d_{20}^{sc}, d_0^{ca}$, there exists a finite number $\mathbf{E}(\phi, d_{10}^{sc}, d_{20}^{sc}, d_0^{ca}) > 0$ such that,

$$\lim_{N \to +\infty} \mathbf{E}\{\sum_{k=0}^{N} \| z(k) \|^2 \mid \phi(.), d_{10}^{sc}, d_{20}^{sc}, d_0^{ca}\} < \mathbf{E}(\phi, d_{10}^{sc}, d_{20}^{sc}, d_0^{ca}) \quad (3.17)$$

holds, where $\mathbf{E}$ is the statistical expectation operator.

3.4.2 NCSs under multiple-packet transmission with completely and partially known transition matrices

In what follows, the sufficient conditions required for the stochastic stability of the closed loop system with completely known transition probability matrices is derived first in **Theorem 3**. Later the case of partially known transition matrices is considered and corresponding controller design is given in **Theorem 4**.

Denote $d_{1k}^{sc} = i \in S_{11}, d_{2k}^{sc} = r \in S_{12}, d_k^{ca} = m \in S_2$, we denote $A(d_k^{ca}) \triangleq A_m$, $B_1(d_{1k}^{sc}, d_k^{ca}) \triangleq B_1(i,m), B_2(d_{2k}^{sc}, d_k^{ca}) \triangleq B_2(r,m)$ and let $k = (I+\Gamma^T\Gamma)^{-1}[\Gamma^T \;\; I]$, $\underline{d_{11}} = min\{d_{1k}^{sc}, k \in \mathbb{Z}i \in S_{11}\}, \underline{d_{12}} = min\{d_{2k}^{sc}, k \in \mathbb{Z}i \in S_{12}\}, \underline{\rho} = min\{\rho_{ii}, i \in S_{11}\}, \underline{\pi} = min\{\pi_{rr}, i \in S_{12}\}$.

Theorem 3:Consider system given in (3.16) with completely known transition probability matrices. The system is said be stochastically stable if there exist matrices $P(i,r,m) > 0, B_1(i,m) > 0, B_2(r,m) > 0, Q_1 > 0, Q_2 > 0, R_1 > 0, R_2 > 0, X_v > 0, v = 1,2,3,4, M_{(i,r,m)(v)}, N_{(i,r,m)(v)}, S_{(i,r,m)(v)}, v = 1,2,3,4,5 \forall i,r \in \mathcal{I}$ such that

$$\begin{bmatrix} -\bar{P}(i,r,m) & 0 & 0 & -\bar{P}(i,r,m) & \bar{P}(i,r,m) & \omega_{(i,r,m)(1)} \\ \bullet & -X_2 & 0 & -\bar{P}(i,r,m) & \bar{P}(i,r,m) & \omega_{(i,r,m)(2)} \\ \bullet & \bullet & -X_1 & -\bar{P}(i,r,m) & \bar{P}(i,r,m) & \omega_{(i,r,m)(3)} \\ \bullet & \bullet & \bullet & -X_4 & -\bar{P}(i,r,m) & \omega_{(i,r,m)(4)} \\ \bullet & \bullet & \bullet & \bullet & -X_3 & \omega_{(i,r,m)(5)} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \omega_{(i,r,m)(6)} \end{bmatrix} < 0 \quad (3.18)$$

where

$$\omega_{(i,r,m)(1)} \triangleq \begin{bmatrix} \bar{P}(i,r,m)A_m & \bar{P}(i,r,m)B_1(i,m) & \bar{P}(i,r,m)B_2(r,m) & \mathbf{O}_{(1\times 8)} \end{bmatrix},$$

$$\omega_{(i,r,m)(2)} \triangleq \begin{bmatrix} \sqrt{d_{11}}X_2(A_m - I) & \sqrt{d_{11}}X_2B_1(i,m) & \mathbf{O}_{(1\times 9)} \end{bmatrix}$$

$$\omega_{(i,r,m)(3)} \triangleq \begin{bmatrix} \sqrt{d_{12}}X_4(A_m - I) & \sqrt{d_{12}}X_4B_2(r,m) & \mathbf{O}_{(1\times 9)} \end{bmatrix}$$

$$\omega_{(i,r,m)(4)} \triangleq \begin{bmatrix} \sqrt{d_{11}}X_1(A_m - I) & \sqrt{d_{11}}X_1B_1(i,m) & \mathbf{O}_{(1\times 9)} \end{bmatrix}$$

$$\omega_{(i,r,m)(5)} \triangleq \begin{bmatrix} \sqrt{d_{12}}X_3(A_m - I) & \sqrt{d_{12}}X_3B_2(r,m) & \mathbf{O}_{(1\times 9)} \end{bmatrix}$$

$$\omega_{(i,r,m)(6)} \triangleq \begin{bmatrix} \varpi_1 & \varpi_2 \end{bmatrix}$$

Where

$$\varpi_1 = \begin{bmatrix} \eta_{(i,r,m)11} & \eta_{(i,r,m)12} & \eta_{(i,r,m)13} & \eta_{(i,r,m)14} & \eta_{(i,r,m)15} & \sqrt{d_{11}}M_{(i,r,m)1} \\ \bullet & \eta_{(i,r,m)22} & \eta_{(i,r,m)23} & \eta_{(i,r,m)24} & \eta_{(i,r,m)25} & \sqrt{d_{11}}M_{(i,r,m)2} \\ \bullet & \bullet & \eta_{(i,r,m)33} & \eta_{(i,r,m)34} & \eta_{(i,r,m)35} & \sqrt{d_{11}}M_{(i,r,m)3} \\ \bullet & \bullet & \bullet & \eta_{(irm)34} & \eta_{(i,r,m)35} & \sqrt{d_{11}}M_{(i,r,m)4} \\ \bullet & \bullet & \bullet & \bullet & \eta_{(i,r,m)35} & \sqrt{d_{11}}M_{(i,r,m)5} \\ \bullet & \bullet & \bullet & \bullet & \bullet & -X_1 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{bmatrix}$$

$$\varpi_2 = \begin{bmatrix} \sqrt{d_{r1}}S_{(i,r,m)1} & \sqrt{d_{11}}N_{(i,r,m)1} & \sqrt{d_{12}}M_{(i,r,m)1} & \sqrt{d_{r2}}S_{(i,r,m)1} & \sqrt{d_{12}}N_{(i,r,m)1} \\ \sqrt{d_{r1}}S_{(i,r,m)2} & \sqrt{d_{11}}N_{(i,r,m)2} & \sqrt{d_{12}}M_{(i,r,m)2} & \sqrt{d_{r2}}S_{(i,r,m)2} & \sqrt{d_{12}}N_{(i,r,m)2} \\ \sqrt{d_{r1}}S_{(i,r,m)3} & \sqrt{d_{11}}N_{(i,r,m)3} & \sqrt{d_{12}}M_{(i,r,m)3} & \sqrt{d_{r2}}S_{(i,r,m)3} & \sqrt{d_{12}}N_{(i,r,m)3} \\ \sqrt{d_{r1}}S_{(i,r,m)4} & \sqrt{d_{11}}N_{(i,r,m)4} & \sqrt{d_{12}}M_{(i,r,m)4} & \sqrt{d_{r2}}S_{(i,r,m)4} & \sqrt{d_{12}}N_{(i,r,m)4} \\ \sqrt{d_{r1}}S_{(i,r,m)5} & \sqrt{d_{11}}N_{(i,r,m)5} & \sqrt{d_{12}}M_{(i,r,m)5} & \sqrt{d_{r2}}S_{(i,r,m)5} & \sqrt{d_{12}}N_{(i,r,m)5} \\ 0 & 0 & 0 & 0 & 0 \\ -X_1 & 0 & 0 & 0 & 0 \\ \bullet & -X_2 & 0 & 0 & 0 \\ \bullet & \bullet & -X_3 & 0 & 0 \\ \bullet & \bullet & \bullet & -X_3 & 0 \\ \bullet & \bullet & \bullet & \bullet & -X_4 \end{bmatrix}$$

$\mathbf{0}_{A_1 \times A_2}$represents the zero matrix of dimension $\mathrm{A}_1 \times \mathrm{A}_2$

Proof: Let the LyapunovKrasovskii functional be

$$\begin{aligned}
V(z(k),k) &= \sum_{s=1}^{9} V_s(z(k),k) \\
V_1(z(k),k) &= z^T(k)P(i,r,m)z(k) \\
V_2(z(k),k) &= \sum_{\tau=k-d_{1k}^{sc}}^{k-1} z^T(\tau)Q_1 z(\tau) \\
V_3(z(k),k) &= \sum_{\tau=k-d_{11}}^{k-1} z^T(\tau)R_1 z(\tau) \\
V_4(z(k),k) &= (1-\underline{\rho}) \sum_{\theta=-d_{11}+1}^{-\underline{d_{11}}} \sum_{\tau=k+\theta}^{k-1} z^T(\tau)Q_1 z(\tau) \\
V_5(z(k),k) &= \sum_{\theta=-d_{11}}^{-1} \sum_{\tau=k+\theta}^{k-1} y^T(\tau)(X_1+X_2)y(\tau) \\
V_6(z(k),k) &= \sum_{\tau=k-d_{2k}^{sc}}^{k-1} z^T(\tau)Q_2 z(\tau) \\
V_7(z(k),k) &= \sum_{\tau=k-d_{12}}^{k-1} z^T(\tau)R_2 z(\tau) \\
V_8(z(k),k) &= (1-\underline{\pi}) \sum_{\theta=-d_{12}+1}^{-\underline{d_{12}}} \sum_{\tau=k+\theta}^{k-1} z^T(\tau)Q_2 z(\tau) \\
V_9(z(k),k) &= \sum_{\theta=-d_{12}}^{-1} \sum_{\tau=k+\theta}^{k-1} y^T(\tau)(X_3+X_4)y(\tau)
\end{aligned}$$

Where $P(i,r,m) = P^T(i,r,m) > 0, R_1 = R_1^T > 0, R_2 = R_2^T > 0, Q_1 = Q_1^T > 0$ and $Q_2 = Q_2^T > 0$ are to be determined. Then, the proof follows a similar procedure as that for the single-packet dropout case; hence, it has been omitted.

Theorem 4: Consider system (3.16) with partially known transition probability matrices. There exists a controller, such that the resulting closed-loop system is stochastically stable if there exist matrices $P(i,r,m) > 0, i,r,m \in I, B_1(i,m) > 0, B_2(r,m) > 0, Q_1, Q_2 > 0, R_1, R_2 > 0, X_v > 0, v = 1,2,3,4, M_{(i,r,m)v}, N_{(i,r,m)v}, S_{(i,r,m)v}, v = 1,2,3,4,5 \forall i,r \in \mathcal{I}$ such that

$$\begin{bmatrix} -\chi(j,q,n) & 0 & 0 & -\bar{P}(i,r,m) & \bar{P}(i,r,m) & \omega_{(i,r,m)(1)} \\ \bullet & -X_2 & 0 & -\bar{P}(i,r,m) & \bar{P}(i,r,m) & \omega_{(i,r,m)(2)} \\ \bullet & \bullet & -X_1 & -\bar{P}(i,r,m) & \bar{P}(i,r,m) & \omega_{(i,r,m)(3)} \\ \bullet & \bullet & \bullet & -X_4 & -\bar{P}(i,r,m) & \omega_{(i,r,m)(4)} \\ \bullet & \bullet & \bullet & \bullet & -X_3 & \omega_{(i,r,m)(5)} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \omega_{(i,r,m)(6)} \end{bmatrix} < 0 \quad (3.19)$$

where

$$\begin{aligned} \omega_{(i,r,m)(1)} \triangleq \; & [\chi(j,q,n)A_m \;\; \chi(j,q,n)B_1(i,m) \;\; \cdots \\ & \chi(j,q,n)B_2(r,m) \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0] \end{aligned}$$

and

$\omega_{(irm)(v)}, v = 2,3,4,5,6$ are as defined in **Theorem 1**.

if $\rho^i_{\mathcal{K}} = 0, \pi^r_{\mathcal{K}} = 0$ and $\lambda^m_{\mathcal{K}} = 0$, then $\chi(j,q,n) \triangleq P(j,q,n)$, otherwise

$$\chi(j,q,n) \triangleq \begin{cases} \frac{1}{\lambda^i_{\mathcal{K}}\rho^m_{\mathcal{K}}\pi^r_{\mathcal{K}}} P^{(i,r,m)}_{\mathcal{K}} \\ \\ P(j,q,n), \forall j \in \mathcal{I}^i_{\mathcal{UK}}, \forall r \in \mathcal{I}^r_{\mathcal{UK}}, \forall n \in \mathcal{I}^m_{\mathcal{UK}} \end{cases}$$

Proof:First of all, we know that the system (3.16) is stochastically stable under the completely known transition probabilities if (3.18) holds. Note that (3.19) can be rewritten as

$$\begin{aligned} \mathbf{E}_{(i,r,m)} \quad &\triangleq \begin{bmatrix} -P^{(i,r,m)}_{\mathcal{K}} & P^{(i,r,m)}_{\mathcal{K}}\mathbf{E}_{(i,r,m)1} \\ \\ \bullet & \lambda^m_{\mathcal{K}}\rho^i_{\mathcal{K}}\pi^r_{\mathcal{K}}\mathbf{E}_{(i,r,m)2} \end{bmatrix} \\ &+ \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}} \sum_{q\in\mathcal{I}^r_{\mathcal{UK}}} \sum_{n\in\mathcal{I}^m_{\mathcal{UK}}} \lambda_{mn}\rho_{ij}\pi_{rq} \\ &\quad \begin{bmatrix} -P(j,q,n) & P(j,q,n)\mathbf{E}_{(i,r,m)1} \\ \bullet & \mathbf{E}_{(i,r,m)2} \end{bmatrix} \end{aligned}$$

where

$$\begin{aligned} \mathbf{E}_{(i,r,m)1} \quad &\triangleq \quad [0 \quad 0 \ \ A_m \quad B_1(i,m) \quad B_2(i,m) \quad B_1(i,m) \quad \cdots \\ &\qquad B_2(r,m) \ \ 0 \ \ 0 \ \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0] \end{aligned}$$

$$\mathbf{E}_{(i,r,m)2} \triangleq \begin{bmatrix} -X_2 & 0 & y & y & \omega_{(i,r,m)(2)} \\ \bullet & -X_1 & y & y & \omega_{(i,r,m)(3)} \\ \bullet & \bullet & -X_4 & y & \omega_{(i,r,m)(4)} \\ \bullet & \bullet & \bullet & -X_3 & \omega_{(i,r,m)(5)} \\ \bullet & \bullet & \bullet & \bullet & \omega_{(i,r,m)(6)} \end{bmatrix}$$

Therefore if one has

$$\begin{bmatrix} -P_{\mathcal{K}}^{(i,r,m)} & P_{\mathcal{K}}^{(i,r,m)}\mathbf{E}_{(irm)1} \\ \\ \bullet & \lambda_{\mathcal{K}}^{m}\rho_{\mathcal{K}}^{i}\pi_{\mathcal{K}}^{r}\mathbf{E}_{(irm)2} \end{bmatrix} < 0 \tag{3.20}$$

$$\begin{bmatrix} -P(j,q,n) & P(j,q,n)\mathbf{E}_{(i,r,m)1} \\ \bullet & \mathbf{E}_{(i,r,m)2} \end{bmatrix} < 0 \tag{3.21}$$

$$\forall j \in \mathcal{I}_{\mathcal{UK}}^{i} \forall q \in \mathcal{I}_{\mathcal{UK}}^{r} \forall n \in \mathcal{I}_{\mathcal{UK}}^{m}$$

then we have $\Xi_{(i,r,m)} < 0$ hence the system is stochastically stable under partially known transition probabilities, which is concluded from the obvious fact that no knowledge on $\rho_{ij}, \pi_{rq}, \forall j \in \mathcal{I}_{\mathcal{UK}}^{i}, \forall q \in \mathcal{I}_{\mathcal{UK}}^{r}$ and $\lambda_{mn} \forall n \in \mathcal{I}_{\mathcal{UK}}^{m}$, is required in (3.20)and (3.21). Thus, for $\rho_{\mathcal{K}}^{i}, \pi_{\mathcal{K}}^{r}, \lambda_{\mathcal{K}}^{m} \neq 0$ and $\rho_{\mathcal{K}}^{i}, \pi_{\mathcal{K}}^{r}, \lambda_{\mathcal{K}}^{m} = 0$,, respectively, one can readily obtain (3.12), since if $\rho_{\mathcal{K}}^{i}, \pi_{\mathcal{K}}^{r}, \lambda_{\mathcal{K}}^{m} = 0$ the conditions (3.20), (3.21) will reduce to (3.20). This completes the proof.

In this case, the control law is

$$
\begin{aligned}
F_{\bar{m}\times G}(d_{1k}^{sc}) &= K\psi_{1,i,0}^{T}Q_1^{-1}\begin{bmatrix} I_G \\ 0_{G\times(\bar{n}-G)} \end{bmatrix} \\
F_{\bar{m}\times(\bar{n}-G)}(d_{2k}^{sc}) &= K\psi_{2,r,0}^{T}Q_2^{-1}\begin{bmatrix} 0_{(\bar{n}-G)\times r} \\ I_{\bar{n}-G} \\ \begin{bmatrix} 0 \end{bmatrix} \end{bmatrix}
\end{aligned} \tag{3.22}
$$

3.5 Numerical Example

Single Packet Transmission

To illustrate the effectiveness of the proposed methodology, We apply the results obtained above to cart and inverted pendulum system [90] as shown in above fig 3.2.

$$
\begin{aligned}
\phi &= \begin{bmatrix} 1 & 0.1 & -.0166 & -.0005 \\ 0 & 1 & -0.3374 & -.0166 \\ 0 & 0 & 1.0996 & 0.1033 \\ 0 & 0 & 2.0247 & 1.0996 \end{bmatrix}, \\
\Gamma &= \begin{bmatrix} .0045 \\ 0.0896 \\ -.0068 \\ -.1377 \end{bmatrix}
\end{aligned}
$$

The eigenvalues of the system are $1, 1, 1.5569$, and 0.6423. Hence, the discrete

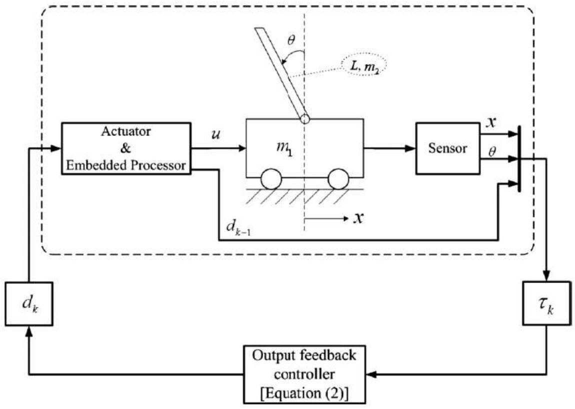

Figure 3.2: Schematic diagram of Cart and inverted pendulum

time system is unstable. Fig. 3.3 shows the open loop response of the above system without any control signal applied.

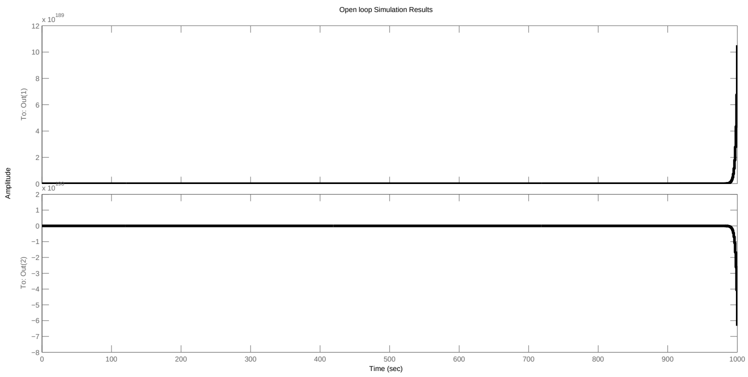

Figure 3.3: Open loop response of Cart and inverted pendulum system

The simulation was carried out on Matlab R2010, wherein, for each given case the LMIs were solved to obtain the gain matrices

For the purpose of simulation, 3 different cases of transition probability matrices were considered as shown below:

In **case [I]** it is been assumed that the transition probability matrices ρ_{ij} and λ_{mn} are completely known.

Consider MJLS with three operation modes. Let the matrices ρ_{ij} and λ_{mn} in completely known case be,

$$\rho_{ij} = \begin{bmatrix} .6 & .3 & .1 \\ .4 & .34 & .26 \\ .22 & .55 & .23 \end{bmatrix}, \quad \lambda_{mn} = \begin{bmatrix} .3 & .4 & .3 \\ .6 & .2 & .2 \\ .5 & .3 & .2 \end{bmatrix}$$

The probabilities ρ_{ij} and λ_{mn} are called transition probabilities. Let us consider the dropouts process in measurement channel .The process can remain in the state it is in, and this occurs with probability ρ_{ii}. An initial probability distribution, defined on S, specifies the starting state. Usually this is done by specifying a particular state as the starting state. For an instance, consider a system with three modes. i.e, the delay in the measurement channel can take any one of these values $S_1 = \{0, 1, 2\}$. From the above given ρ_{ij} matrix, we see that if their is a delay of zero at first transition cycle then the event that their would be a delay of two from now is the disjoint union of the following three events 1) Their is a delay of zero at next transition cycle and a delay of two after two transition cycle from now 2) Their is a delay of one at next transition cycle and a delay of two after two transition cycle from now, and 3) Their is a delay of two at next transition cycle and also after two transition cycle from now. The probability of the first of these events is the product of the conditional probability that their is a delay of zero at next transition cycle, given that their is a delay of zero at current instant, and the conditional probability that their is a delay of two after two transition cycle from now, given that their is a delay of zero at next transition cycle. Using the transition matrix , we can write this product as $\rho_{11}\rho_{13}$. The other two events also have probabilities that can be written as products of entries of ρ. Thus, we have

$$\rho_{13} = \rho_{11}\rho_{13} + \rho_{12}\rho_{23} + \rho_{13}\rho_{33}$$

This equation is similar to the dot product of two vectors, where the first row of ρ is dotted with the third column of ρ. Similarly the probability matrix λ_{mn} is inferred.

Then by using *Theorem 2* we obtain the controller gains F_i as

$$\begin{aligned}
F_0 &= \begin{bmatrix} 0.00576 & -0.01543 & -0.00654 & -0.00005 \end{bmatrix} \\
F_1 &= \begin{bmatrix} -0.00086 & -0.00801 & -0.00034 & -0.0015 \end{bmatrix} \\
F_2 &= \begin{bmatrix} -0.00057 & -0.00076 & -0.0003 & -0.0014 \end{bmatrix}
\end{aligned}$$

By **Theorem 2**, we can obtain the control law as

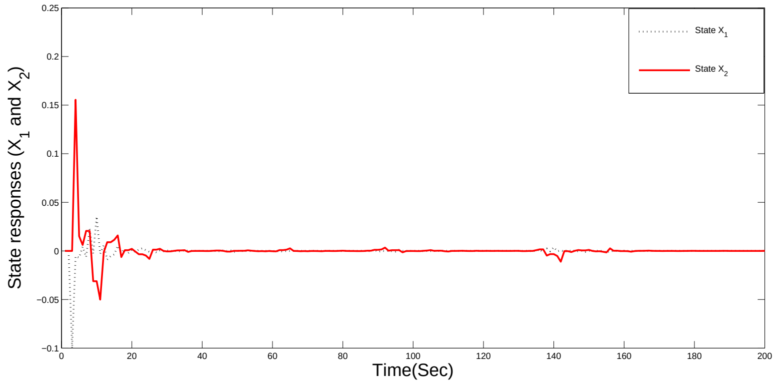

Figure 3.4: Case I: Response curves of state variable x_d and $\dot{x}_d$ in cart and inverted pendulum system

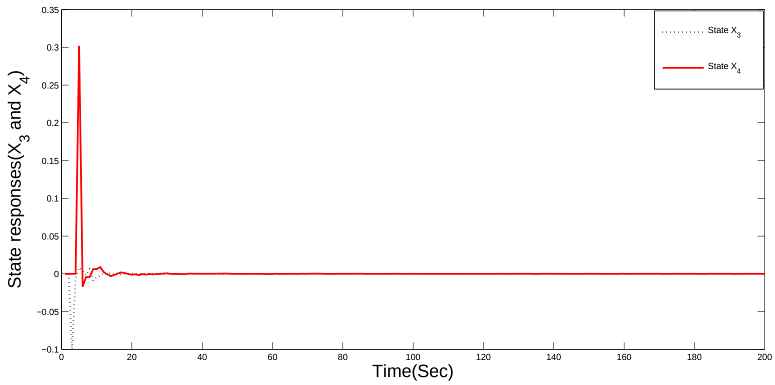

Figure 3.5: Case I: Response curves of state variable θ and $\dot{\theta}$ in cart and inverted pendulum system

In **case** [**II**] it is been assumed that the transition probability matrices ρ_{ij} and λ_{mn} are partially known. Some of the elements in these matrices are unknown and missing even though they are time invariant.

Let the matrices ρ_{ij} and λ_{mn} in partially known case be,

$$\rho_{ij} = \begin{bmatrix} ? & ..6 & ? \\ .4 & .6 & 0 \\ .2 & ? & ? \end{bmatrix}, \quad \lambda_{mn} = \begin{bmatrix} .4 & ? & ? \\ .4 & ? & ? \\ .5 & .3 & .2 \end{bmatrix}$$

$$\begin{aligned} F_0 &= \begin{bmatrix} -0.00054 & -0.04365 & -0.00065 & -0.00066 \end{bmatrix} \\ F_1 &= \begin{bmatrix} -0.000122 & -0.00207 & -0.0034 & -0.0001 \end{bmatrix} \\ F_2 &= \begin{bmatrix} -0.00099 & -0.00022 & -0.00495 & -0.0029 \end{bmatrix} \end{aligned}$$

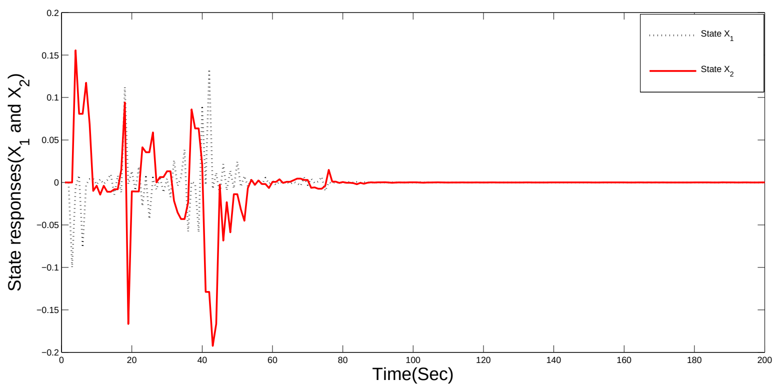

Figure 3.6: Case II: Response curves of state variable x_d and $\dot{x}_d$ in cart and inverted pendulum system

In **Case [III]** also it is been assumed that the transition probability matrices ρ_{ij} and λ_{mn} are partially known. But the number of unknown elements in these

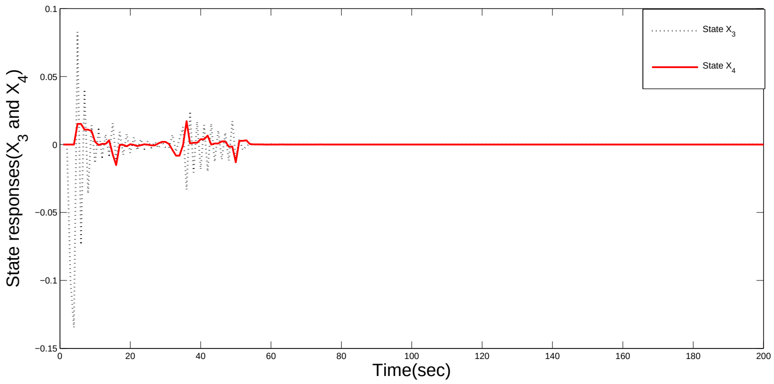

Figure 3.7: Case II: Response curves of state variable θ and $\dot{\theta}$ in cart and inverted pendulum system

matrices are more when compared to case [II].

Let the matrices ρ_{ij} and λ_{mn} in case[III] be,

$$\rho_{ij} = \begin{bmatrix} ? & ? & ? \\ .35 & .6 & 0.05 \\ ? & ? & ? \end{bmatrix}, \quad \lambda_{mn} = \begin{bmatrix} .65 & .15 & .2 \\ ? & ? & ? \\ .7 & ? & ? \end{bmatrix}$$

$$\begin{aligned} F_0 &= \begin{bmatrix} -0.0061 & -0.000366 & -0.000603 & -0.0006322 \end{bmatrix} \\ F_1 &= \begin{bmatrix} -0.000122 & -0.000883 & -0.0022 & -0.0009125 \end{bmatrix} \\ F_2 &= \begin{bmatrix} -0.000572 & -0.000196 & -0.00991 & -0.000418 \end{bmatrix} \end{aligned}$$

From the above figures it can clearly seen that the time taken for the states of the system to settle to zero is more in case III when compared to other two cases.

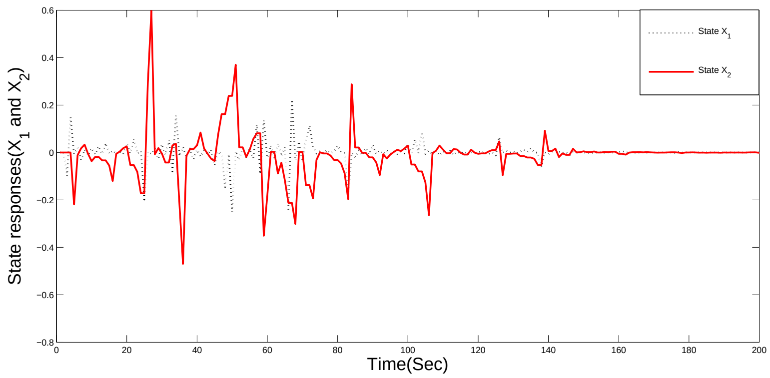

Figure 3.8: Case III: Response curves of state variable x_d and $\dot{x}_d$ in cart and inverted pendulum system

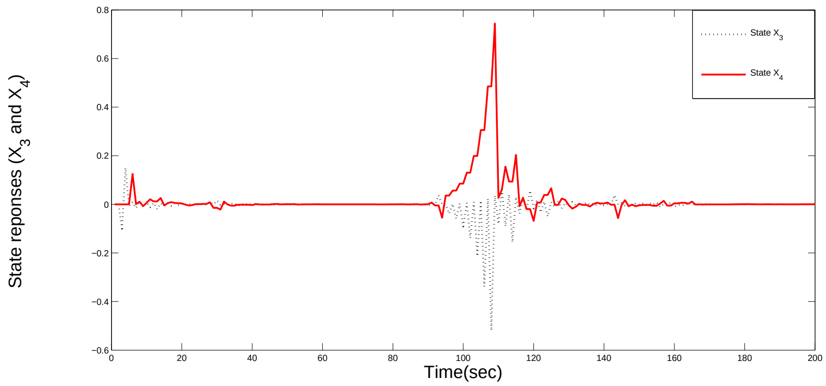

Figure 3.9: Case III: Response curves of state variable θ and $\dot{\theta}$ in cart and inverted pendulum system

That is, the time taken by the system in Case III to bring the pendulum in the upright position is more when compared to the other two cases. This is because, more the information we have about the transition probabilities, better is the performance of the system. This shows a tradeoff between cost of obtaining the transition probabilities and system performance.

Multi Packet Transmission

For simplicity, we suppose $d_{1k}^{sc} \in \{0, 1, 2\}, d_{2k}^{sc} \in \{0, 1, 2\}$ and $d_k^{ca} \in \{0, 1, 2\}$. There are two channels, namely, $X_1 = x_1(k)$ and $X_2 = x_2(k)$. Their transition probability matrices for d_{1k}^{sc}, d_{2k}^{sc} and d_k^{ca} are

$$\rho_{ij} = \begin{bmatrix} ? & .4 & ? \\ .5 & .6 & 0.05 \\ .22 & ? & ? \end{bmatrix}, \quad \pi_{ij} = \begin{bmatrix} .3 & .4 & .3 \\ ? & ? & 0.0765 \\ .22 & ? & ? \end{bmatrix}$$

$$\lambda_{mn} = \begin{bmatrix} .5 & .? & .? \\ .64 & ? & ? \\ .7 & ? & ? \end{bmatrix}$$

By Theorem 4, we can obtain the control law as

$$\begin{aligned}
F_{10} &= \begin{bmatrix} -0.0044 & -0.0006 & -0.003 & -0.0063 \end{bmatrix} \\
F_{11} &= \begin{bmatrix} -0.0001 & -0.0003 & 0.0022 & -0.0016 \end{bmatrix} \\
F_{12} &= \begin{bmatrix} -0.007 & -0.0016 & 0.0091 & 0.00418 \end{bmatrix} \\
F_{20} &= \begin{bmatrix} 0.00128 & -0.0009 & -0.0022 & -0.009125 \end{bmatrix} \\
F_{21} &= \begin{bmatrix} 0.00179 & 0.0081 & -0.0002 & -0.0053 \end{bmatrix} \\
F_{22} &= \begin{bmatrix} -0.0053 & -0.0098 & -0.0022 & -0.0025 \end{bmatrix}
\end{aligned}$$

The state trajectories versus the time curves are shown in Fig. 3.10 and 3.11,

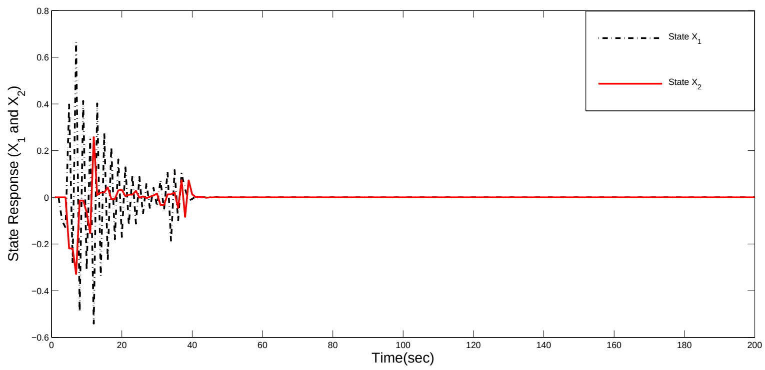

Figure 3.10: Response curve of state variable x_d and $\dot{x}_d$ in cart and inverted pendulum system

which illustrates that the controllers we designed can guarantee the stochastic stability of the NCSs with the multiple-packet transmissions. As in single packet transmission, similar analysis can be carried out for multi-packet NCSs with

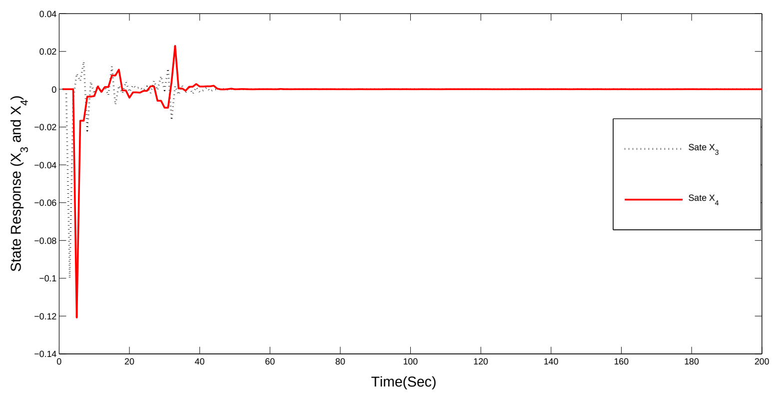

Figure 3.11: State Response curve of state variables θ and $\dot{\theta}$ in cart and inverted pendulum system

different cases of transition probability matrices.

3.6 Conclusions

This paper provides a state feedback stabilization technique for Markov jump linear networked control systems subject to network phenomena such as packet dropouts on S/C and C/A side. The packets dropout on S/C and C/A side are modeled by two independent Markov chains. Some of the elements of transition probability matrices are considered to be unknown which cover the general from of known matrices. Stability criterion was based on the Lyapunov-functional theory. Numerical simulations are presented to illustrate the effectiveness of developed techniques. We look forward to extending our results to robust control problem by incorporating time varying elements of the transition matrices.

Chapter 4

OBSERVER BASED NETWORKED PREDICTIVE CONTROL FOR MJLS

4.1 Introduction

IN the recent years, we are witnessing in many systems such as traffic, communication, aviation and spaceflight that the task of controllers design is based on communication networks. Particularly, due to enormous use of internet these days, the trend of network based systems is on rapid growth. This benefit the control systems in many ways including retrieving data from the plant, to responding to fluctuations in the plant from anywhere around the world at any time. A control system in which sensors and actuators communicate with each other over a communication network is termed as *a distributed real time control system*. It offers advantage of modularity and flexibility in system design. The structure of NCSs is different from that of traditional control systems, because of the presence of various specific problems exist in NCSs. These include loss of data packets, network delays, network security and safety, to name a few. Therefore, a special care must be taken while handling this wide class of systems. Much research has been witnessed in the recent years to address the issues related to networked control systems [14]-[13]

Even though a wide range of research has recently been reported in the area of networked control systems, many researchers failed to answer some of the important and practical issues related to networked control systems like the communication delays, which are varying in a random fashion and the issue of packet dropouts. An important feature of networked control system which is being mostly ignored by the researcher is that, communication network has the ability to transfer a packet of data set at the same time, which cannot be accomplished in traditional control system. In this paper, we take full opportunity of this NCSs feature and proposed a novel dynamic output feedback based networked predictive control technique to deal with network control systems with data packets dropouts in both measurement channel (sensor -controller S/C) and actuation channel (controller-actuator-C/A).

On another research front in the past few decades, model predictive control

(MPC), has received much attention from the researchers in dealing with the problems of NCSs like, transmission losses or packet losses. Extensive application of MPC controller is reported in the control of many industrial processes like, textile , rubber, steel , plastic, and so on. In [76] a nonlinear model predictive control (NMPC) is demonstrated in a framework of general event-based/ asynchronous systems. It was shown that NCSs suffering from random delays and information losses can be made to work in a stable environment by a proposed NMPC technique. The key to guarantee asymptotic convergence is the selection of large prediction horizon value. In [88], an expanded investigation was done by adopting nonlinear model predictive control tool on the stability of networked control systems. The 'set invariance' was then suggested, which mainly deals with the stabilization of constrained robust state feedback uncertain discrete-time systems. Robust control policy was combined with model predictive control scheme to take into consideration various network defects like model uncertainty, time-varying transmission delays and and packet dropouts which effects the system stability and performance. In [42], the authors studied the plant state behavior between transmission times, and suggested an explicit model to estimate this plant states. Model-based networked control systems (MB-NCSs) were investigated in detail and stability condition was derived. Three feedback network communication models were presented in [53] and came up with a modified MPC technique to handle the delays and dropouts in the measurement and actuation channel of NCSs. Predicted sequence of future control signals are used to compensate for the delay in the forward communication channel. Another important contribution made was to analyze the stability criteria for both, fixed and random communication time delays. In [43] a model predictive control was proposed for network control systems with time stamping feature added to it. In their analysis, the authors considered that the communication delays introduced by the networks are of random nature bounded with in minimum and maximum values. The proposed time-stamping with a buffer improves the performance and stability of the network over wider range of delay. Experimental validations were provided to support the proposed technique. In [106], an explicit model predictive controller was proposed to estimate the packet dropout in Networked Control Systems (NCS).

In [70], a new predictive control strategy was developed for networked control systems. The design considerations were such that the infinite horizon quadratic objective is reduced at each sampling time, while ensuring the stochastic stability of the closed-loop system. In their design, the first control value of the most recently received signal is implemented directly to the plant by the actuator. Markovian chains are used to describe the networked communication delays which are assumed to be random and bounded. Linear matrix inequality (LMI) approach was used to derive the delay-dependent conditions and existence of controller. In their analysis it was assumed that the states are directly available from the plant. In practicality however, the states are not readily available from the plant to be used by the controller. They are to be estimated. So, the investigation carried out in [70] was without the consideration of observer or the state estimator. The work presented in this paper is an extension of the analysis carried out in [70] by taking into consideration an observer to estimate the states and the addition of buffer in the actuation channel to store the data received from the plant and by considering various other additional factors. Such type of considerations further improve the analysis in terms of system performance versus the packet loss.

In most of the design procedure of model predictive controllers, the state $x(k_i)$ is assumed to be available at the time k_i. But in practice, with most applications, the state $x(k_i)$ at the time k_i is not available all the time. At some instances, the measure of state $x(k_i)$ becomes impossible. The soft instrument used to estimate unknown state variables based on process measurement, is called an observer. In this paper, we make use of the observer to estimate the states $x(k_i)$ and proposed a new networked predictive control scheme which can overcome the effects of data dropout in network modeled as Markovian channels.

The networked predictive control scheme proposed in this paper, consists of a controller which is designed to generate a set of future prediction. These control predictions at time k are packed and sent to the plant through a network, which are then stored in the buffer(B2) placed between controller and actuator. On the actuator side only the latest control prediction set is stored and the first control value from this prediction set is applied to the plant. Similarly a buffer(B1) is placed in the measurement channel to take care of the data dropouts occurring during the transmission of packets from sensor to observer. These methods play a very important role in dealing with dropouts in the proposed NCS implementation.

The main features of this paper over the earlier works are:

1. Firstly, we considered packet dropouts both in the network connecting plant and observer and also in the network connecting observer and controller which is more realistic and practical in nature.
2. Secondly, the controller gain(F) and observer gain(L) are decided based on the number of packet dropped out on the sensor controller side(d_k^{sc}).
3. The stability analysis for the proposed networked predictive control scheme is also performed for the class of discrete-time Markov jump systems with partially known transition matrices.

According to author's knowledge, such kind of analysis has so far not fully addressed in the literature of networked model predictive control. Therefore the contribution of this paper is twofold:

- It mainly deals with the design of predictive networked control scheme for an NCSs such that infinite horizon quadratic objective is reduced at each sampling instant, while ensuring the stability of the closed-loop system.
- It takes into consideration, the dropouts of the packets on both sensor to controller(S/A) side and controller and actuator(C/A) side modeled using underlying Markovian chain. The stability analysis is performed by first assuming that the transition probability matrices describing these packets

dropouts on both the side S/C and C/A respectively are completely known and then the case of partially known transition matrices is considered.

The remaining part of this paper is organized as follows:Section II mainly deals with the problem description and formulation which is helpful throughout this paper and gives the basic idea of our setup. In section III, the stability analysis is investigated for both completely known and unknown transition probabilities cases and the controller design is discussed for closed loop system. Section IV presents the illustrative example to exploit the effectiveness of the proposed methodology. Finally in section V, we conclude the work.

Notations: The notations given in this paper are fairly standard. The super script $''T''$ stands for matrix transposition, $\mathbb{R}^n$ denotes the n dimensional Euclidean space. $\mathbb{N}$ represents the set of natural numbers. The $diag\{\cdots\}$ represents the block diagonal matrix. I and 0 stand for identity matrix and zero matrix respectively. $\mathbf{E}$ denotes the expectation operator with some probability measure. $\bullet$ represents the symmetric terms in the block matrix. Matrices if they are not explicitly specified, are assumed to have compatible dimensions.

4.2 Problem Formulation

Consider the NCS setup as shown in Fig 4.1. This NCS setup is assumed to suffer from transmission delay possibly due to the packets dropout at S/C and

C/A side. Here the sensors, controllers, and actuators are all clock-driven. The linear time-invariant (LTI) plant we consider here is:

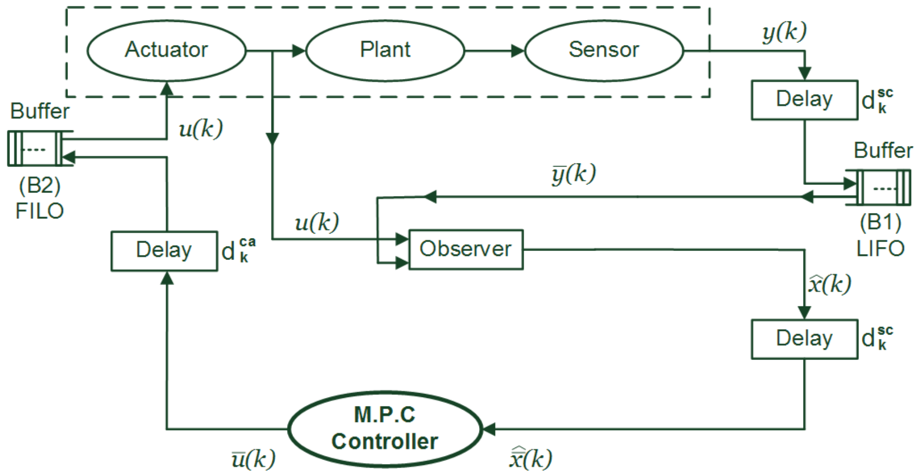

Figure 4.1: The setup of Networked Predictive Control scheme

$$
\begin{aligned}
x(k+1) &= \phi x(k) + \Gamma u(k) \\
y(k) &= \sigma x(k)
\end{aligned}
\tag{4.1}
$$

where $x(k) \in \Re^n$ represents the plant states , $u(k) \in \Re^m$ and $y(k) \in \Re^p$ represents the plant's control input and output vectors, respectively. ϕ, Γ and σ are known real constant matrices with appropriate dimensions.

The buffer (B1) in S/C channel is supposed to be long enough to hold all the output packets arrived from the sensor, which will be picked up by observer according to the last-in-first-out rule. For example, during the transmission from sensor to observer, if the sensor data $y(k)$ is lost, the observer will read out the most recent data $y(k-1)$ from the buffer(B1) and utilize it as $\bar{y}(k)$ to estimate the new state variable $\hat{x}$, which will be transmitted to the controller on a network to calculate the new control input $\bar{u}(k)$; otherwise, the new sensor data $y(k)$ will be saved to the buffer(B1) and used by the observer as $\bar{y}(k)$. These predicted states $\hat{x}$ are transmitted to the controller through a network which again suffer from random delay (d_k^{sc}) modeled using underlying Markov chain. Thus, for the buffer(B1), we have

$$\bar{y}(k) = \begin{cases} y(k) & \text{if successfully transmitted} \\ \bar{y}(k-1) & \text{otherwise} \end{cases} \tag{4.2}$$

Similarly, buffer (B2) in C/A channel is assumed to be large and sufficient to stack all the data arrived from the controller and that the delay established by the buffer(B2) can be neglected when inflected with the delay induced by the network. From this assumption, it can be concluded that the most recently received control signal will used by actuator. Therefore the buffer (B2) in C/A channel works according to the rule of, first-in-last-out(FILO). Assume that the control predictions sequences $[\bar{u}^T(k|k), \bar{u}^T(k+1|k), \cdots, \bar{u}^T(k+N-1|k), \cdots, \bar{u}^T(k+N|k)]^T$ calculated at time k are packed and sent to the plant through a network are successfully transmitted, then on the actuator side, first control signal $u(k|k)$ of latest control prediction sequence is used as control input to the plant. At the next time instant, if package containing predictive sequences are delayed or dropped, then one step prediction $u(k+1|k)$ will be selected by actuator to be used as the control input to the plant. If their is a successful transmission of predictive sequences package on (C/A) side, then first control input of this newly received prediction sequence package will be used by actuator otherwise, $u(k+2|k)$ will be used by actuator$\cdots$ and so on. Thus, for the buffer(B2), we have

$$u(k|k) = \begin{cases} \bar{u}(k|k) & \text{if successfully transmitted} \\ \bar{u}(k+d_k^{ca}|k) & \text{otherwise} \end{cases} \tag{4.3}$$

When the full state information is not available, it is desirable to design the following observer-based controller:

$$\begin{aligned} \hat{x}(k+1|k) &= \phi\hat{x}(k|k) + \Gamma u(k|k) \\ &+ L(d_k^{sc})(\bar{y}(k|k) - \hat{y}(k|k)) \\ \hat{y}(k) &= \sigma\hat{x}(k) \end{aligned} \tag{4.4}$$

Consider the state feedback control law as

$$\bar{u}(k|k) = F(d_k^{sc})\hat{\hat{x}}(k|k) \tag{4.5}$$

where $F(d_k^{sc})$ and $L(d_k^{sc})$ are the set of controllers and observer gains, designed based on d_k^{sc}.

Remark 4.2.1 *It is to emphasized that d_k^{sc} and d_k^{ca} are the quantities of packets dropped at time k on the S/C and C/A sides, respectively.*

The basic principle of MPC controller is to study and predict the effect of independent variables($u(k)$) on dependent variables($y(k)$) in the modeled system. For this purpose, MPC make use of plant measurements($x(k|k)$), dynamic state of the process, the MPC models, and the process variable targets. These predictions are calculated by satisfying the constraints on both ($u(k)$) and ($y(k)$). It works on the principle of receding horizon control, where only the first change in each independent variable($u(k|k)$) is implemented, and in the next change same calculations are repeated. The control predictions are calculated as

$$\bar{u}(k+q|k) = F(d_k^{sc})\hat{\hat{x}}(k+q|k)$$

and the future control trajectory is denoted by

$$[\bar{u}^T(k|k), \bar{u}^T(k+1|k), \cdots, \bar{u}^T(k+q-1|k), \cdots, \ \ \bar{u}^T(k+q|k)]^T$$

The predicted states are constructed as

$$\begin{aligned}
\hat{\hat{x}}(k+1|k) &= (\phi + \Gamma F(d_k^{sc}) - L(d_k^{sc}))\hat{\hat{x}}(k|k) \\
&+ \sigma L(d_k^{sc})\hat{\hat{x}}(k|k)) \\
\hat{\hat{x}}(k+2|k) &= \phi\hat{\hat{x}}(k+1|k) + \Gamma u(k+1|k)) \\
\vdots \ &\vdots \ \vdots \\
\hat{\hat{x}}(k+p|k) &= \phi\hat{\hat{x}}(k+p-1|k) + \Gamma u(k+q-1|k))
\end{aligned}$$

Therefore the future state variables are denoted as

$$[\hat{\hat{x}}(k+1|k), \hat{\hat{x}}(k+2|k), \cdots, \hat{\hat{x}}(k+N-1|k), \cdots, \ \ \hat{\hat{x}}(k+p|k)]$$

where $x(k+p|k)$ is the predicted state variable at $k+p$ with given current plant information $x(k|k)$.

From the predicted state variables, the predicted output variables are, by substitution

$$
\begin{aligned}
\hat{\hat{y}}(k+1|k) &= \sigma\phi\hat{x}(k|k) + \sigma\Gamma u(k|k) \\
\hat{\hat{y}}(k+2|k) &= \sigma\phi^2\hat{x}(k|k) + \sigma\phi\Gamma u(k|k) + \sigma\Gamma u(k+1|k) \\
\hat{\hat{y}}(k+3|k) &= \sigma\phi^3\hat{x}(k|k) + \sigma\phi^2\Gamma u(k|k) + \sigma\phi\Gamma u(k+1|k) \\
&+ \sigma\Gamma u(k+2|k) \\
\vdots &\ \vdots \\
\hat{\hat{y}}(k+p|k) &= \sigma\phi^p\hat{x}(k|k) + \sigma\phi^{p-1}\Gamma u(k|k) \\
&+ \sigma\phi^{p-2}\Gamma u(k+1|k) + \cdots + \cdots \\
&+ \sigma\phi^{p-q}\gamma u(k+q-1|k).
\end{aligned}
$$

Therefore the future predicted outputs are denoted as

$$
[\hat{\hat{y}}(k+1|k), \hat{\hat{y}}(k+2|k), \cdots, \hat{\hat{y}}(k+N|k), \cdots, \ \ \hat{\hat{y}}(k+p|k)]
$$

Remark 4.2.2 *In the foregoing analysis, it can be seen that all predicted variables are expressed in terms of current state variable $x(k|k)$ and the future control movement $u(k|k+q)$, where $j = 0, 1, \cdots, q-1$. In the sequel, p, q are assumed to be lengths of prediction and control horizon sequences, respectively.*

The number of packets dropped on the S/C side at time k is assume to be d_k^{sc}, which is calculated from the current time k to the last successful transmission (happened at time $k - d_k^{sc}$), d_k^{ca} is the packet dropped quantity on the C/A side from the current time k and last successful transmission at time $k - d_k^{ca}$, and both d_k^{sc} and d_k^{ca} are assumed to be bounded. Therefore we have,

$$0 \leq d_k^{sc} \leq d_1, \;\; 0 \leq d_k^{ca} \leq d_2$$

where d_1 and d_2 are considered to be positive integer. Two homogeneous independent Markov chains are used to model d_k^{sc} and d_k^{ca}, which take values in $S_1 = \{0, 1, \cdots, d_1\}$ and $S_2 = \{0, 1, \cdots, d_2\}$.

Their transition probabilities are defined by

$$\begin{aligned} \rho_{ij} &= Pr(d_{k+1}^{sc} = j | d_k^{sc} = i) \\ \lambda_{rq} &= Pr(d_{k+1}^{ca} = q | d_k^{ca} = r) \end{aligned} \tag{4.6}$$

where $\rho_{ij} = 0, i, j \in S_1, \lambda_{rq} \geq 0, m, n \in S_2$ and $\sum_{j=0}^{d_1} \rho_{ij} = 1, \sum_{q=0}^{d_2} \lambda_{rq} = 1$. It is obvious that the transition probabilities satisfy

$$\begin{aligned}
\rho_{ij} &\geq 0, \quad if \;\; j \neq i+1 \;\; and \;\; j \neq 0 \\
\lambda_{rq} &\geq 0, \quad if \;\; q \neq m+1 \;\; and \;\; q \neq 0 \\
\rho_{ij} &= \begin{bmatrix} \rho_{11} & \rho_{12} \cdots \rho_{1N} \\ \rho_{21} & \rho_{22} \cdots \rho_{2N} \\ & \\ \rho_{N1} & \rho_{N2} \cdots \rho_{NN} \end{bmatrix}, \quad \lambda_{rq} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \cdots \lambda_{1N} \\ \lambda_{21} & \lambda_{22} \cdots \lambda_{2N} \\ & \\ \lambda_{N1} & \lambda_{N2} \cdots \lambda_{NN} \end{bmatrix}
\end{aligned}$$

Consider plant (4.1), where the data packets containing the control sequences are assumed to be successfully transmitted. Substituting (4.3) and (4.5) into system in (4.1), we have the following closed loop system.

$$\begin{aligned}
x(k+1|k) &= \phi x(k|k) + \Gamma\{F(d_k^{sc})\}\hat{\hat{x}}(k|k) \\
x(k+1|k) &= \phi x(k|k) + \Gamma\{F(d_k^{sc})\}\hat{x}(k - d_k^{sc}|k) \\
x(k+1|k) &= \phi x(k|k) + \Gamma\{F(d_k^{sc})\}x(k - d_k^{sc}|k) \\
&- \Gamma\{F(d_k^{sc})\}e(k - d_k^{sc}|k) \qquad (4.7)
\end{aligned}$$

Again, consider plant (4.1), with the assumption that the transmission of data packets containing the control sequences is not successful. Substituting (4.3) and (4.5) into system into (4.1), we have the following closed loop system.

$$\begin{aligned}
x(k+1|k) &= \phi x(k|k) + \Gamma\bar{u}(k + d_k^{ca}|k) \\
x(k+1|k) &= \phi x(k|k) + \Gamma\{F(d_k^{sc})\}\hat{\hat{x}}(k + d_k^{ca}|k) \\
x(k+1|k) &= \phi x(k|k) + \Gamma\{F(d_k^{sc})\}\hat{x}(k - (d_k^{sc} - d_{k-d_k^{sc}}^{ca})|k) \\
x(k+1|k) &= \phi x(k|k) + \Gamma\{F(d_k^{sc})\}x(k - (d_k^{sc} - d_{k-d_k^{sc}}^{ca})|k) \\
&- \Gamma\{F(d_k^{sc})\}e(k - (d_k^{sc} - d_{k-d_k^{sc}}^{ca}|k)
\end{aligned} \tag{4.8}$$

Remark 4.2.3 *It is significant by simple manipulations to infer that*

1. $\hat{\hat{x}}(k|k) = \hat{x}(k - d_k^{sc}|k)$, *which can be easily derived by iterations based on (4.2).*
2. $e(k) = x(k) - \hat{x}(k)$
3. $\hat{\hat{x}}(k) = \hat{x}(k - d_k^{sc})$

State observer given in (4.4) with no delay in either of the channels is represented as

$$\begin{aligned}
\hat{x}(k+1|k) &= \phi\hat{x}(k|k) + \Gamma\{F(d_k^{sc})\}\hat{\bar{x}}(k|k) \\
&+ \{L(d_k^{sc})\}(\bar{y}(k) - \hat{y}(k)) \\
\hat{x}(k+1|k) &= \phi\hat{x}(k|k) + \Gamma\{F(d_k^{sc})\}\hat{x}(k - d_k^{sc}|k) \\
&+ \{L(d_k^{sc})\}(y(k|k) - \hat{y}(k|k)) \\
\hat{x}(k+1|k) &= \phi\hat{x}(k|k) + \Gamma\{F(d_k^{sc})\}x(k - d_k^{sc}|k) \\
&- \Gamma\{F(d_k^{sc})\}e(k - (d_k^{sc})|k) \\
&+ \sigma\{L(d_k^{sc})\}e(k)|k)
\end{aligned} \tag{4.9}$$

Again, by considering the state observer given in (4.4) with delay in both the channel, is represented as,

$$\begin{aligned}
\hat{x}(k+1|k) &= \phi\hat{x}(k|k) + \Gamma\bar{u}(k + d_k^{ca}|k) \\
&+ L(d_k^{sc})(\bar{y}(k) - \hat{y}(k)) \\
\hat{x}(k+1|k) &= \phi\hat{x}(k|k) + \Gamma F(d_k^{sc})\hat{\bar{x}}(k + d_k^{ca}|k) \\
&+ L(d_k^{sc})(y(k - d_k^{sc} - 1|k) - \hat{y}(k - d_k^{sc} - 1|k)) \\
\hat{x}(k+1|k) &= \phi\hat{x}(k|k) + \Gamma F(d_k^{sc})\hat{x}(k - (d_k^{sc} - d_{k-d_k^{sc}}^{ca})|k) \\
&+ L(d_k^{sc})(y(k - d_k^{sc} - 1) - \hat{y}(k - d_k^{sc} - 1))
\end{aligned}$$

$$\begin{aligned}
\hat{x}(k+1|k) &= \phi\hat{x}(k|k) + \Gamma F(d_k^{sc})x(k-(d_k^{sc}-d_{k-d_k^{sc}}^{ca})|k) \\
&- \Gamma F(d_k^{sc})e(k-(d_k^{sc}-d_{k-d_k^{sc}}^{ca})|k) \\
&+ \sigma L(d_k^{sc})e(k-d_k^{sc}-1)|k)
\end{aligned} \quad (4.10)$$

To simplify the expression of the closed-loop system obtained in (4.7) and (4.8), we introduce a function $\alpha(\cdot)$ to yield the closed-loop system as

$$\begin{aligned}
x(k+1|k) &= \phi x(k|k) + (1-\alpha(d_k^{ca}))\Gamma F(d_k^{sc})x(k-d_k^{sc}|k) \\
&- (1-\alpha(d_k^{ca}))\Gamma F(d_k^{sc})e(k-d_k^{sc}|k) \\
&+ \alpha(d_k^{ca})\Gamma F(d_k^{sc})x(k-(d_k^{sc}-d_{k-d_k^{sc}}^{ca})|k) \\
&- \alpha(d_k^{ca})\Gamma F(d_k^{sc})e(k-(d_k^{sc}-d_{k-d_k^{sc}}^{ca})|k)
\end{aligned} \quad (4.11)$$

where

$$\alpha(d_k^{ca}) = \begin{cases} 1 & d_k^{ca} > 0 \\ 0 & d_k^{ca} = 0 \end{cases}$$

Let the estimation error be defined as

$$e(k|k) = x(k|k) - \hat{x}(k|k)$$

By subtracting (4.9) from (4.7) and (4.10) from (4.8), we get the error equation as

$$\begin{aligned} e(K+1|k) &= \phi e(k|k) - \sigma L(d_k^{sc})e(k|k) \\ e(K+1|k) &= \phi e(k|k) - \sigma L(d_k^{sc})e(k - d_k^{sc} - 1|k) \end{aligned} \tag{4.12}$$

Error equations obtained in the above case can be combined with a function $\beta(d_k^{sc})$, to get

$$\begin{aligned} e(K+1|k) &= \phi e(k|k) - (1 - \beta(d_k^{sc}))\sigma L(d_k^{sc})e(k|k) \\ &- (\beta(d_k^{sc}))\sigma L(d_k^{sc})e(k - d_k^{sc} - 1|k) \end{aligned} \tag{4.13}$$

where

$$\beta(d_k^{sc}) = \begin{cases} 1 & d_k^{sc} > 0 \\ \\ 0 & d_k^{sc} = 0 \end{cases}$$

By concatenating the plant (4.11) and error (4.13) vectors, we obtain a global vector as $z(k) = [x^T(k|K) \quad e^T(k|K)]^T$. Therefore the closed-loop system for the NCS with packet dropout in both S/C and C/A channels is represented as shown in (4.14).

$$\begin{aligned}
z(k+1|k) &= \begin{bmatrix} \phi & 0 \\ 0 & (\phi - (1-\beta(d_k^{sc}))\sigma L(d_k^{sc})) \end{bmatrix} \begin{bmatrix} x(k|k) \\ e(k|k) \end{bmatrix} \\
&+ \begin{bmatrix} (1-\alpha(d_k^{ca}))\Gamma F(d_k^{sc}) & -(1-\alpha(d_k^{ca}))\Gamma F(d_k^{sc}) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(k-d_k^{sc}|k) \\ e(k-d_k^{sc}|k) \end{bmatrix} \\
&+ \begin{bmatrix} \alpha(d_k^{ca})\Gamma F(d_k^{sc}) & -\alpha(d_k^{ca}))\Gamma F(d_k^{sc}) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(k-(d_k^{sc} - d_{k-d_k^{sc}}^{ca})|k)) \\ e(k-(d_k^{sc} - d_{k-d_k^{sc}}^{ca})|k) \end{bmatrix} \\
&+ \begin{bmatrix} 0 & 0 \\ 0 & -\beta(d_k^{sc})\sigma L(d_k^{sc}) \end{bmatrix} \begin{bmatrix} x(k-d_k^{sc}-1|k) \\ e(k-d_k^{sc}-1|k) \end{bmatrix} \\
z(k+1|k) &= A(d_k^{sc})z(k|k) + B(d_k^{sc}, d_k^{ca})z(k-(d_k^{sc})|k) + C(d_k^{sc})z(k-(d_k^{sc} - d_{k-d_k^{sc}}^{ca})|k) \\
&+ D(d_k^{sc})z(k-(d_k^{sc}-1)|k) \qquad (4.14)
\end{aligned}$$

The resulting closed-loop system in (4.14) is a jump linear system with two modes (d_k^{sc}) and (d_k^{ca}) and one mode-dependent time-varying delay (d_k^{sc}), where their transitions are described by two Markov chains, which give the history behavior of S/C and C/A packet dropouts, respectively. This also enables us to apply the results of jumping linear systems with time-delays to the analysis and synthesis of such NCSs.

4.2.1 Networked predictive control scheme

In order to minimize nominal cost $J_p(k)$ over a prediction horizon p, it is assumed that q control steps $u(k+m|k), m = 0, 1, \cdots, q-1$, are calculated. Therefore the nominal cost $J_p(k)$ is given as [70]:

$$min_{u(k+m|k), m=0,1,\cdots,q-1} J_p(k) \tag{4.15}$$

The case of infinite horizon is considered in this paper, which means, $p = q = \infty$. Therefore the quadratic objective is [70]

$$\begin{aligned} J_\infty(k) &= \sum_{m=0}^{\infty} \mathbf{E}\{z(k+m|k)^T Q z(k+m|k) \\ &+ u(k+m|k)^T R u(k+m|k)\} \end{aligned} \tag{4.16}$$

where $Q > 0, R \geq 0$ are symmetric matrices.

Consider the model of the linear time-invariant discrete plant as in (4.14) and assume that at each sampling time k, the system's output accessible is $y(k|k) = y(k)$. Therefore the scheme for controller design at each sampling time k can be formulated as follows:

1. measure the output $y(k|k)$;
2. compute the state-feedback gain $F(d_k^{sc})$ and observer gain $L(d_k^{sc})$, such that the performance objective in (4.16) is minimized.
3. Finally, apply the first control move $u(k|k)$ of the latest predicted control sequence.

It can also be inferred from the plant model in (4.14) and step 2., that the predicted state $z(k+m|k)$ satisfy the following difference equation

$$\begin{aligned} z(k+m+1|k) &= A(d_k^{sc})z(k+m|k) + B(d_k^{sc}, d_k^{ca})z(k+m-(d_{k+m}^{sc})|k) \\ &+ C(d_k^{sc})z(k+m-(d_{k+m}^{sc} - d_{k+m-d_{k+m}^{sc}}^{ca})|k) \\ &+ D(d_k^{sc})z(k+m-d_{k+m}^{sc}-1|k) \end{aligned} \tag{4.17}$$

Finding a procedure to figure out the optimization issue in step 2 at every sampling instant k is a key to solve the MPC problem. In the following, we give the sufficient conditions for the γ-suboptimal problem

$$J_{\infty}(k) \leq \gamma \tag{4.18}$$

for a given $\gamma > 0$

4.3 Main Result

4.3.1 Stability Analysis And Controller Design with Completely Known Transition Probability Matrices

In the following section, the sufficient conditions required for the stability analysis and controller synthesis problems for networked predictive control with completely Known transition probability matrices in (4.14) is discussed. It is considered that the transition probability matrices ρ and λ are completely available for the analysis. Firstly, stability analysis and the derivation for the sufficient condition, for which the closed-loop networked predictive control system (4.14) with the given controller (4.5) and (4.3) is exponentially stable in the mean square is performed.

For notational simplicity, in the sequel, for $d_k^{sc} = i \in S_1, d_k^{ca} = r \in S_2$, we denote $A(d_k^{sc}) \triangleq A(i)$,$B(d_k^{sc}, d_k^{ca}) \triangleq B(i,r)$, $C(d_k^{sc}, d_k^{ca}) \triangleq C(i,r)$, $D(d_k^{sc}) \triangleq D(i)$ and $\underline{d_1} = min\{d_k^{sc}, k \in \mathbb{Z} \in S_1\}, \underline{d_2} = min\{d_k^{ca}, k \in \mathbb{Z} \in S_2\}, \underline{\rho} = min\{\rho_{ii}, i \in S_1\}$,

$\kappa = 1 + (1 - \underline{\rho})(d_1 - \underline{d}_1)$ and $\underline{\lambda} = min\{\lambda_{rr}, r \in S_2\}$

Let the Lyapunov-Krasovskii functional be

$$V(z(k+m|k) = \sum_{s=1}^{7} V_s z(k+m|k) \tag{4.19}$$

with

$$\begin{aligned}
V_1(z(k+m|k)) &= z^T(k+m|k)p(i,r)z(k+m|k) \\
V_2(z(k+m|k)) &= \sum_{\tau=k+m|k-d^{sc}_{k+m|k}}^{k+m|k-1} z^T(k+m|k)Qz(k+m|k) \\
V_3(z(k+m|k)) &= \sum_{\tau=k+m|k-(d^{sc}_{k+m|k}-d^{ca}_{k+m-d^{sc}_{k+m|k}})}^{k+m|k-1} z^T(k+m|k)Qz(k+m|k) \\
V_4(z(k+m|k)) &= \sum_{\tau=k+m|k-(d^{sc}_{k+m|k})-1|k}^{k+m|k-1} z^T(k+m|k)Qz(k+m|k) \\
V_5(z(k+m|k)) &= (\kappa) \sum_{\theta=-d_1+2}^{-\underline{d}_1+1} \sum_{\tau=k+m+\theta-1}^{k+m-1} z^T(k+m|k)Qz(k+m|k) \\
V_6(z(k+m|k)) &= (1+\kappa) \sum_{\theta=-(d_1-d_2)+2}^{-(\underline{d}_1-\underline{d}_2)+1} \sum_{\tau=k+m+\theta-1}^{k+m-1} z^T(k+m|k)Qz(k+m|k) \\
V_7(z(k+m|k)) &= (1+\kappa) \sum_{\theta=-d_1+3}^{-\underline{d}_1} \sum_{\tau=k+m+\theta-1}^{k+m-1} z^T(k+m|k)Qz(k+m|k)
\end{aligned}$$

Where $P(i,r) = P^T(i,r) > 0$ and $Q = Q^T > 0$ are to be determined. At the sampling time k, suppose that the following inequality holds for all $z(k+m|k)$ and $u(k+m|k)$, $m \geq 0$ satisfying (4.14)

$$\begin{aligned}
&\mathbf{E}\{V(z(k+m+1|k) - \mathbf{E}\{V(z(k+m|k)\} \\
&\leq -\mathbf{E}\{z(k+m|k)^T Q z(k+m|k) \\
&+u(k+m|k)^T R u(k+m|k)\}
\end{aligned} \tag{4.20}$$

The quadratic performance index J_∞ is finite, only when $\mathbf{E}\{V(z(\infty|k)\} = 0$. Therefore, from (4.20), we obtain

$$-V(z(k|k)) \;\; \leq \;\; -J_\infty(k) \tag{4.21}$$

The following Lemma [70]-[99] is introduced which is useful in deriving the sufficient conditions for the NCS stability:

Lemma 3.1: Choose $V(z(k|k))$ as in (4.19). If there exist real scalars $\delta \geq 0, \mu > 0, v > 0$, and $0 < \omega < 1$ such that

$$\begin{aligned}
\mu\|z(k|k)\|^2 \;\; &\leq \;\; V(z(k|k)) \leq v\|z(k|k)\|^2 \\
\mathbf{E}\{V(z(k+1)|z(k))\} \;\; &- \;\; V(z(k)|k) \leq \delta - \omega V(z(k)|k)
\end{aligned}$$

then the sequence $z(k)$ satisfies

$$\mathbf{E}\{\|z(k)|k\|^2\} \leq \frac{v}{\mu}\|z(0)\|^2(1-\omega)^k + \frac{\delta}{\mu\omega}$$

Theorem 3.1: Let the controller and the observer gain matrices, $F(d_k^{sc})$ and $L(d_k^{sc})$ be given. The closed-loop system (4.14) is said to be exponentially mean square stable if there exist symmetric positive-definite matrices $P(i,r)$¿0, $Q > 0, \Psi_{i,r}$, and matrices $R_u, S_u, M_u, u = 1, 2, 3, \Phi_{1u(i,r)}, u = 1, 2, 3, 4, 5, 6, 7$ such that the matrix inequality (4.22) holds,

$$\mathbf{E}_{(i,r)} \triangleq \begin{bmatrix} \varpi_1 & \varpi_2 \end{bmatrix} < 0 \tag{4.22}$$

Where

$$\varpi_1 = \begin{bmatrix} -\Phi_{11(i,r)} + \Psi_{i,r} & -R_1 + S_1^T & -R_2 + S_2^T & -R_3 + S_3^T \\ \bullet & -\Pi_{22} & 0 & -S_1 - M_1^T \\ \bullet & \bullet & -S_2 - S_2^T - Q & 0 \\ \bullet & \bullet & \bullet & -\Pi_{33} \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{bmatrix}$$

$$\varpi_2 = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ 0 & 0 & 0 \\ -S_1 - M_1^T & 0 & 0 \\ 0 & -S_3 - M_3^T & 0 \\ \Pi_{34} & \Pi_{35} & 0 \\ \bullet & -\Pi_{55} & \Pi_{56} \\ \bullet & \bullet & -\Pi_{66} \end{bmatrix}$$

$$\begin{aligned}
\Pi_{11} &= -R_1 + M_1^T - \Phi_{12}, \;\; \Pi_{12} = -R_2 + M_2^T - \Phi_{13(i,r)} \\
\Pi_{13} &= -R_3 + M_3^T - \Phi_{14(i,r)}, \;\; \Pi_{22} = S_1 + S_1^T + Q \\
\Pi_{33} &= S_3 + S_3^T + Q, \;\; \Pi_{34} = -M_1 - M_1^T + \Phi_{15(i,r)} \\
\Pi_{35} &= \mathbf{E}(B(i,r)^T \bar{P}(i,r) C(i,r)), \;\; \Pi_{55} = M_2 + M_2^T - \Phi_{16(i,r)} \\
\Pi_{56} &= \mathbf{E}(C(i,r)^T \bar{P}(i,r) D(i)), \;\; \Pi_{66} = M_3 + M_3^T - \Phi_{17(i)} \\
\Psi_{i,r} &= -P(i,r) + (d_1 - d_2 + \underline{d}_1 - \underline{d}_2 + 2)Q \\
&+ \;\; R_1 + R_2 + R_3 + R_1^T + R_2^T + R_3^T \\
\Phi_{11(i,r)} &= \mathbf{E}\big\{(A(i) + B(i,r) + C(i,r) + D(i))^T \bar{P}(i,r) \\
&\;\; (A(i) + B(i,r) + C(i,r) + D(i))\big\}
\end{aligned}$$

$$\begin{aligned}
\Phi_{12(i,r)} &= \mathbf{E}\big\{(A(i)+B(i,r)+C(i,r)+D(i))^T\bar{P}(i,r) \\
&\quad B(i,r)\big\} \\
\Phi_{13(i,r)} &= \mathbf{E}\big\{(A(i)+B(i,r)+C(i,r)+D(i))^T\bar{P}(i,r) \\
&\quad C(i,r)\big\} \\
\Phi_{14(i,r)} &= \mathbf{E}\big\{(A(i)+B(i,r)+C(i,r)+D(i))^T\bar{P}(i,r) \\
&\quad D(i)\big\} \\
\Phi_{15(i,r)} &= \mathbf{E}\big\{B(i,r)^T\bar{P}(i,r)B(i,r)\big\} \\
\Phi_{16(i,r)} &= \mathbf{E}\big\{C(i,r)^T\bar{P}(i,r)C(i,r)\big\} \\
\Phi_{17(i)} &= \mathbf{E}\big\{D(i)^T\bar{P}(i,r)D(i)\big\}
\end{aligned}$$

$\bar{P}(i,r)=\sum_{q=0}^{d_2}\sum_{j=0}^{d_1}\lambda_{rq}\rho_{ij}P(j,q)$

Proof: Defining $y(k+m|k)=x(k+m+1|k)-x(k+m|k)$, one has

$$z(k+m|k-d_k^{sc}) = z(k+m|k) - \sum_{\tau=k+m-d_{k+m}^{sc}|k}^{k+m|k-1} y(\tau|k) \tag{4.23}$$

$$z(k+m|k-(d_k^{sc}-d_{k-d_k^{sc}}^{ca})) = z(k+m|k) - \sum_{\tau=k+m-(d_{k+m}^{sc}-d_{k+m-d_{k+m}^{sc}}^{ca})|k}^{k+m|k-1} y(\tau|k) \tag{4.24}$$

$$z(k+m|k-d_k^{sc}-1) = z(k+m|k) - \sum_{\tau=k+m-d_{k+m}^{sc}-1|k}^{k+m|k-1} y(\tau) \tag{4.25}$$

Then, system (4.14) can be transformed into

$$\begin{aligned} z(k+1|k) &= (A(i)+B(i,r)+C(i,r)+D(i))z(k|k) \\ &- B(i,r)\eta(k|k)-C(i,r)\psi(k|k)-D(i)\zeta(k|k) \qquad (4.26) \end{aligned}$$

where

$$\begin{aligned} \eta(k+m|k) &= \sum_{\tau=k+m-d^{sc}_{k+m}}^{k+m-1} y(\tau|k) \\ \psi(k+m|k) &= \sum_{\tau=k+m-(d^{sc}_{k+m}-d^{ca}_{k+m-d^{sc}_{k+m}})}^{k+m-1} y(\tau|k) \\ \zeta(k+m|k) &= \sum_{\tau=k+m-d^{sc}_{k+m}-1}^{k+m-1} y(\tau|k) \end{aligned}$$

By taking the difference and mathematical expectation of $V_1(z(k+m|k))$ along the system (4.26), we have

$$\begin{aligned} & \mathbf{E}\{\Delta V_1(z(k+m|k))\} \\ =\ & \mathbf{E}\{V_1(z(k+m+1|k))-V_1(z(k+m|k)\} \\ =\ & z^T(k+m|k)[\Phi_{11(i,r)}-P(i,r)]z(k+m|k) \\ -\ & 2z^T(k+m|k)\Phi_{12(i,r)}\eta(k+m|k) \end{aligned}$$

$$
\begin{aligned}
&- \quad 2z^T(k+m|k)\Phi_{13(i,r)}\psi(k+m|k) \\
&- \quad 2z^T(k+m|k)\Phi_{14(i,r)}\zeta(k+m|k) \\
&+ \quad \eta^T(k+m|k)\Phi_{15(i,r)}\eta(k+m|k) \\
&+ \quad \psi^T\Phi_{16(i,r)}\psi(k+m|k) + \zeta^T\Phi_{17(i)}\zeta(k+m|k) \\
&+ \quad 2\eta^T(k+m|k)\mathbf{E}\{B(i,r)^T\bar{P}(i,r)C(i,r)\}\psi(k+m|k) \\
&+ \quad 2\psi^T(k+m|k)\mathbf{E}\{C(i,r)^T\bar{P}(i,r)D(i)\}\zeta(k+m|k)
\end{aligned}
$$

$$
\begin{aligned}
& \mathbf{E}\{\Delta V_2(z(k+m|k))\} \\
= \quad & \sum_{s=k+m+1-d^{sc}_{k+m+1}}^{k+m} z^T(s)Qz(s) \\
- \quad & \sum_{s=k+m-d^{sc}_{k+m}}^{k+m-1} z^T(s)Qz(s) \\
= \quad & z^T(k+m)Qz(k+m) \\
- \quad & z^T(k+m-d^{sc}_{k})Qz(k+m-d^{sc}_{k+m}) \\
+ \quad & \sum_{s=k+m+1-d^{sc}_{k+m+1}}^{k+m-1} z^T(s)Qz(s) \\
- \quad & \sum_{s=k+m+1-d^{sc}_{k+m}}^{k+m-1} z^T(s)Qz(s)
\end{aligned}
$$

Note that

$$
\begin{aligned}
&\sum_{s=k+m+1-d^{sc}_{k+m+1}}^{k+m-1} z^T(s)Qz(s) \\
= &\sum_{s=k+m+1-d^{sc}_{k+m+1}}^{k+m-d^{sc}_{k+m}} z^T(s)Qz(s) \\
+ &\sum_{s=k+m+1-d^{sc}_{k+m}}^{k+m-1} z^T(s)Qz(s) \\
\leq &\sum_{s=k+m+1-d^{sc}_{k+m}}^{k+m-1} z^T(s)Qz(s) \\
+ &\sum_{s=k+m+1-d_1}^{k+m-\underline{d}_1} z^T(s)Qz(s)
\end{aligned}
$$

Therefore

$$
\begin{aligned}
&\mathbf{E}\{\Delta V_2(z(k+m|k)) \leq z^T(k+m|k)Qz(k+m|k) \\
- \;&z^T(k+m|k-d^{sc}_{k+m|k})Qz(k+m|k-d^{sc}_{k+m|k}) \\
+ &\sum_{s=k+m+1-d_1}^{k+m-\underline{d}_1} z^T(s)Qz(s)
\end{aligned}
$$

Similarly $\mathbf{E}\{\Delta V_3(z(k+m|k))$ and $\mathbf{E}\{\Delta V_4(z(k+m|k))$ are computed in the same way as $\mathbf{E}\{\Delta V_2(z(k+m|k))$, and it yields

$$\begin{aligned}
& \mathbf{E}\{\Delta V_3(z(k+m|k)) \leq z^T(k+m|k)Qz(k+m|k) \\
- \quad & z^T(k+m-(d^{sc}_{k+m}-d^{ca}_{k+m-d^{sc}_{k+m}})|k) \\
& Qz(k+m-(d^{sc}_{k+m}-d^{ca}_{k+m-d^{sc}_{k+m}})|k) \\
+ \quad & \sum_{s=k+m+1-(d_1-d_2)}^{k+m-(\underline{d}_1-\underline{d}_2)} z^T(s)Qz(s) \\
& \mathbf{E}\{\Delta V_4(z(k+m|k)) \leq z^T(k+m|k)Qz(k+m|k) \\
- \quad & z^T(k+m|k-(d^{sc}_{k+m|k})-1|k) \\
& Qz(k+m|k-(d^{sc}_{k+m|k})-1|k) \\
+ \quad & \sum_{s=k+m+2-d_1}^{k+m-\underline{d}_1} z^T(s)Qz(s)
\end{aligned}$$

Finally

$$\begin{aligned}
& \mathbf{E}\{\Delta V_5(z(k+m|k)) \\
= \quad & \kappa \sum_{\theta=-d_1+2}^{-\underline{d}_1+1} [z^T(k+m|k)Qz(k+m|k) \\
- \quad & z^T(k+m+\theta-1|k)Qz(k+m+\theta-1)] \\
= \quad & (1-\underline{\rho\lambda})[(d_1-\underline{d}_1)z^T(k+m|k)Qz(k+m|k) \\
- \quad & \sum_{s=k+m+1-d_1}^{k+m-\underline{d}_1} z^T(s)Qz(s)]
\end{aligned}$$

$$\begin{aligned}
& \mathbf{E}\{\Delta V_6(z(k+m|k)) \\
= \;& (1+\kappa)[((d_1-d_2)-(\underline{d}_1-\underline{d}_2))z^T(k+m|k)Qz(k+m|k) \\
- \;& \sum_{s=k+m+1-(d_1-d_2)}^{k+m-(\underline{d}_1-\underline{d}_2)} z^T(s)Qz(s)]
\end{aligned}$$

$$\begin{aligned}
& \mathbf{E}\{\Delta V_7(z(k+m|k)) \\
= \;& (1+\kappa)[((d_1-1)-(\underline{d}_1-1)z^T(k+m|k)Qz(k+m|k) \\
- \;& \sum_{s=k+m+2-d_1}^{k+m-(\underline{d}_1-1)} z^T(s)Qz(s)]
\end{aligned}$$

The following conditions are satisfied according to (4.23), (4.24) and (4.25):

$$\begin{aligned}
z(k+m|k)-z(k+m|k-d_k^{sc}) &= 0 \\
z(k+m|k)-z(k+m|k-(d_k^{sc}-d_{k-d_k^{sc}}^{ca})) &= 0 \\
z(k+m|k)-z(k+m|k-d_k^{sc}-1) &= 0
\end{aligned} \tag{4.27}$$

Therefore, for any matrices R_u, S_u, and $M_u, i = 1, 2, , 3$ with appropriate dimensions, we have the following equations:

$$\begin{aligned} & 2[z^T(k+m)R_1 + z^T(k+m-d^s_{k+m}c)S_1 + \eta(k+m)M_1)] \\ \times \quad & [z(k+m) - z(k+m-d^s_{k+m}c) - \eta(k+m)] = 0 \end{aligned} \tag{4.28}$$

$$\begin{aligned} & 2[z^T(k+m)R_2 + z^T(k+m-(d^{sc}_{k+m} - d^{ca}_{k+m-d^{sc}_{k+m}})|k)S_2 \\ + \quad & \psi(k+m)M_2)] \\ \times \quad & [z(k+m) - z(k+m-(d^{sc}_{k+m} - d^{ca}_{k+m-d^{sc}_{k+m}})|k) - \\ & \psi(k+m)] = 0 \end{aligned} \tag{4.29}$$

$$\begin{aligned} & 2[z^T(k+m)R_3 + z^T(k+m-(d^{sc}_{k+m|k}) - 1|k)S_3 \\ + \quad & \zeta(k+m)M_3)] \\ \times \quad & [z(k+m) - z(k+m-(d^{sc}_{k+m|k}) - 1|k) \\ - \quad & \zeta(k+m)] = 0 \end{aligned} \tag{4.30}$$

By adding all the Lyapunov terms, we have

$$
\begin{aligned}
&= \sum_{s=1}^{7} \mathbf{E}\{V(z(k+m|k))\} \\
&\leq z^T(k+m)(\Phi_{11(i,r)} + \Psi_{(i,r)})z(k+m) \\
&+ z^T(k+m)(-2R_1 + 2S_1^T)z(k+m-d^{sc}_{k+m}) \\
&+ z^T(k+m)(-2R_2 + 2S_2^T) \\
&\quad z(k+m-(d^{sc}_{k+m} - d^{ca}_{k+m-d^{sc}_{k+m}})|k) \\
&+ z^T(k+m)(-2R_3 + 2S_3^T)z(k+m-(d^{sc}_{k+m}-1)|k) \\
&+ z^T(k+m)(-2R_1 + 2M_1^T - 2\Phi_{12(i,r)})\eta(k+m) \\
&+ z^T(k+m)(-2R_2 + 2M_2^T - 2\Phi_{13(i,r)})\phi(k+m) \\
&+ z^T(k+m)(-2R_3 + 2M_3^T - 2\Phi_{14(i,r)})\zeta(k+m) \\
&+ z^T(k+m-d^{sc}_{k+m})(-S_1 - S_1^T - Q)z(k+m-d^{sc}_{k+m}) \\
&+ z^T(k+m-d^{sc}_{k+m})(-2S_1 - 2M_1^T)\eta(k+m) \\
&+ z^T(k+m-(d^{sc}_{k+m} - d^{ca}_{k+m-d^{sc}_{k+m}})|k) \\
&\quad (-S_2 - S_2^T - Q)z(k+m-(d^{sc}_{k+m} - d^{ca}_{k+m-d^{sc}_{k+m}})|k) \\
&+ z^T(k+m-(d^{sc}_{k+m} - d^{ca}_{k+m-d^{sc}_{k+m}})|k)(-2S_2 - 2M_2^T) \\
&\quad \psi(k+m) \\
&+ z^T(k+m-d^{sc}_{k+m}-1)(-S_3 - S_3^T - Q) \\
&\quad z(k+m-d^{sc}_{k+m}-1) \\
&+ z^T(k+m-d^{sc}_{k+m}-1)(-2S_3 - 2M_3^T)\zeta(k+m) \\
&+ \eta^T(k+m)(-M_1 - M_1^T + \Phi_{15(i,r)})\eta(k+m) \\
&+ \phi^T(k+m)(-M_2 - M_2^T + \Phi_{16(i,r)})\phi(k+m)
\end{aligned}
$$

$$\begin{aligned}
&+ \quad \zeta^T(k+m)(-M_3 - M_3^T + \Phi_{17(i)})\zeta(k+m) \\
&+ \quad \eta^T(k+m)(\mathbf{E}\{B(i,r)^T \bar{P}(i,r) C(i,r)\})\phi(k+m) \\
&+ \quad \phi^T(k+m)(\mathbf{E}\{C(i,r)^T \bar{P}(i,r) D(i)\})\zeta(k+m) \\
&= \quad \epsilon^T(k+m)\Lambda\epsilon(k+m) \qquad (4.31)
\end{aligned}$$

where $\epsilon(k+m)$, the extended vector is,

$$\epsilon(k+m) = \Big[\; z^T(k+m|k) \quad z^T(k+m-(d^{sc}_{k+m})|k) \quad z^T(k+m-(d^{sc}_{k+m} - d^{ca}_{k+m-d^{sc}_{k+m}})|k) \quad \cdots \\
z^T(k+m-d^{sc}_{k+m}-1|k) \quad \eta^T(k+m|k) \quad \phi^T(k+m|k) \quad \zeta^T(k+m|k) \; \Big]^T$$

If $\Lambda < 0$ holds, i.e., matrix inequality(4.22) holds, then

$$\begin{aligned}
& \mathbf{E}\{V(z(k+m+1)|z(k)),\cdots,z(0)\} - V(z(k+m)|z(k)) \\
= \quad & \epsilon^T(k+m)\Lambda\epsilon(k+m) \\
\leq \quad & -\delta_{min}(-\Lambda)\epsilon^T(k+m)\epsilon(k+m) \\
< \quad & -\nu\epsilon^T(k+m)\epsilon(k+m) \qquad (4.32)
\end{aligned}$$

Where

$$0 < \nu < min\{\delta_{min}(-\Lambda), \digamma\}$$

$$\digamma = max\{\delta_{max}(P(i,r), \delta_{max}(Q)\}$$

Inequality (4.27) implies that

$$\begin{aligned} & \mathbf{E}\{V(z(k+m+1)|z(k)), \cdots, z(0)\} - V(z(k+m)|z(k)) \\ < \quad & \nu\epsilon^T(k+m)\epsilon(k+m) < \frac{-\nu}{\digamma}V(z(k+m)) \end{aligned} \tag{4.33}$$

Which implies that,

$$V(z(k|k)) \leq J_\infty(k)$$

From (4.18), we can conclude that,

$$V(z(k|k)) \leq \gamma \tag{4.34}$$

This completes the proof.

Now, the following theorem gives a solution to the problem of the observer-based stabilizing controller design.

Theorem 3.2: For given scalars $d_1, d_2, \underline{d}_1, \underline{d}_2$, $\alpha(d_k^{ca})$ and $\beta(d_k^{sc})$, the closed-loop system (4.14) is exponentially mean square stable if there exist matrices $P(i,r)^T = P(i,r) > 0, Q = Q^T > 0, \Psi_{i,r} > 0, X^T = X > 0, Q > 0, R_u, S_u$ and $M_u, u = 1,2,3$ and real matrices $F_i \in \Re^{m\times n}$ and $L_i \in \Re^{n\times p}$ such that we have inequalities (4.35) and (4.36)

$$\mathbf{E}_{(i,r)} \triangleq \begin{bmatrix} \varpi_1 & \varpi_2 \end{bmatrix} < 0 \tag{4.35}$$

$$P(i,r)X(i,r) = I \tag{4.36}$$

where

$$\varpi_1 = \begin{bmatrix} -\Psi_{i,r} & -R_1 + S_1^T & -R_2 + S_2^T & -R_3 + S_3^T \\ \bullet & -S_1 - S_1^T - Q & 0 & -S_1 - M_1^T \\ \bullet & \bullet & -S_2 - S_2^T - Q & 0 \\ \bullet & \bullet & \bullet & -S_3 - S_3^T - Q \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{bmatrix}$$

$$\varpi_2 = \begin{bmatrix} -R_1 + M_1^T & -R_2 + M_2^T & \Pi_{17} & \Pi_{18} \\ 0 & 0 & 0 & 0 \\ -S_2 - M_2^T & 0 & 0 & 0 \\ 0 & -S_3 - M_3^T & 0 & 0 \\ -M_1 - M_1^T & 0 & \Pi_{57} & 0 \\ \bullet & -M_2 - M_2^T & \Pi_{67} & 0 \\ \bullet & \bullet & \Pi_{77} & 0 \\ \bullet & \bullet & \bullet & -X \end{bmatrix}$$

$$\begin{aligned} \Pi_{17} &= -R_3 + M_3^T, \quad \Pi_{18} = \Theta_{12}^T F_i^T \Theta_{13}^T + \Theta_{14}^T L_i^T \Theta_{15}^T \\ \Pi_{57} &= -\beta(d_k^{sc}) L_i^T \Theta_{52}^T, \quad \Pi_{67} = -\alpha(d_k^{ca}) F_i^T \Theta_{15}^T \\ \Pi_{77} &= -M_2 - M_2^T \end{aligned}$$

$$\Theta_{11} = \begin{bmatrix} \phi & 0_{n\times n} \\ 0_{n\times n} & \phi \end{bmatrix}, \Theta_{12} = \begin{bmatrix} I_{n\times n} \\ -I_{n\times n} \end{bmatrix}^T$$

$$\Theta_{13} = \begin{bmatrix} \Gamma \\ 0_{n\times m} \end{bmatrix}, \Theta_{14} = \begin{bmatrix} 0_{p\times n} & \sigma \end{bmatrix}, \Theta_{15} = \begin{bmatrix} 0_{n\times n} \\ -I_{n\times n} \end{bmatrix}$$

Proof: By the Schur complement, (4.22) is equivalent to (4.37) as shown below.

$$\mathbf{E}_{(i,r)} \triangleq \begin{bmatrix} \varpi_1 & \varpi_2 \end{bmatrix} < 0 \tag{4.37}$$

where

$$\varpi_1 = \begin{bmatrix} -\Psi_{i,r} & -R_1 + S_1^T & -R_2 + S_2^T & -R_3 + S_3^T \\ \bullet & -S_1 - S_1^T - Q & 0 & -S_1 - M_1^T \\ \bullet & \bullet & -S_2 - S_2^T - Q & 0 \\ \bullet & \bullet & \bullet & -S_3 - S_3^T - Q \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{bmatrix}$$

$$
\varpi_2 = \begin{bmatrix} -R_1 + M_1^T & -R_2 + M_2^T & \Pi_{17} & \Pi_{18} \\ 0 & 0 & 0 & 0 \\ -S_2 - M_2^T & 0 & 0 & 0 \\ 0 & -S_3 - M_3^T & 0 & 0 \\ -M_1 - M_1^T & 0 & 0 & \Pi_{58} \\ \bullet & -M_2 - M_2^T & 0 & \Pi_{68} \\ \bullet & \bullet & \Pi_{77} & \Pi_{78} \\ \bullet & \bullet & \bullet & \Pi_{78} \end{bmatrix}
$$

$$
\begin{aligned}
\Pi_{17} &= -R_3 + M_3^T \quad \Pi_{18} = \mathbf{E}\{(A(i) + B(i,r) + C(i,r) + D(i))^T\} \\
\Pi_{58} &= -M_3 - M_3^T \quad \Pi_{68} = -\mathbf{E}\{B(i,r)^T\} \quad , \Pi_{88} = -P^{-1}(i,r) \\
\Pi_{77} &= -\mathbf{E}\{C(i,r)^T\} \quad \Pi_{78} = -\mathbf{E}\{D(i,r)^T\}
\end{aligned}
$$

The expressions $\mathbf{E}\{(A(i)+B(i,r)+C(i,r)+D(i))\}^T, \mathbf{E}\{B(i,r)\}^T, \mathbf{E}\{C(i,r)\}^T$ and $\mathbf{E}\{D(i)\}^T$, are obtain as,

$$
\begin{aligned}
&\mathbf{E}\{(A(i) + B(i,r) + C(i,r) + D(i))\}^T \\
&= \begin{bmatrix} \phi + \Gamma F(i) & -\Gamma F(i) \\ 0 & \phi - \sigma L(i) \end{bmatrix}^T
\end{aligned}
$$

$$\mathbf{E}\{B(i,r)\}^T = \begin{bmatrix} (1-\alpha(d_k^{ca}))\Gamma F(i) & -(1-\alpha(d_k^{ca}))\Gamma F(i) \\ 0 & 0 \end{bmatrix}^T$$

$$\mathbf{E}\{C(i,r)\}^T = \begin{bmatrix} \alpha(d_k^{ca})\Gamma F(i) & -\alpha(d_k^{ca}))\Gamma F(i) \\ 0 & 0 \end{bmatrix}^T$$

$$\mathbf{E}\{D(i)\}^T = \begin{bmatrix} 0 & 0 \\ 0 & -\beta(d_k^{sc})\sigma L(i) \end{bmatrix}^T$$

In order to transform matrix inequality (4.36) into an LMI, $\mathbf{E}\{(A(i)+B(i,r)+C(i,r)+D(i))\}^T, \mathbf{E}\{B(i,r)\}^T, \mathbf{E}\{C(i,r)\}^T$ and $\mathbf{E}\{D(i)\}^T$ can be represented as,

$$\begin{aligned} & \mathbf{E}\{(A(i)+B(i,r)+C(i,r)+D(i))\}^T \\ = & \begin{bmatrix} \phi^T & 0_{n\times n} \\ 0_{n\times n} & \phi^T \end{bmatrix} \\ + & \begin{bmatrix} I_{n\times n} \\ -I_{n\times n} \end{bmatrix} F^T(i) \begin{bmatrix} \Gamma^T & 0_{n\times n} \end{bmatrix} \\ + & \begin{bmatrix} I_{n\times n} \\ \sigma_{p\times n} \end{bmatrix} L^T(i) \begin{bmatrix} 0_{n\times n} & -I_{n\times n} \end{bmatrix} \end{aligned} \tag{4.38}$$

$$
\begin{aligned}
& \mathbf{E}\{B(i,r)\}^T \\
= \ & (1-\alpha(d_k^{ca})) \begin{bmatrix} \Gamma^T \\ 0_{n\times m} \end{bmatrix} F^T(i) \begin{bmatrix} I_{n\times n} & -I_{n\times n} \end{bmatrix}
\end{aligned} \tag{4.39}
$$

$$
\begin{aligned}
& \mathbf{E}\{C(i,r)\}^T \\
= \ & (\alpha(d_k^{ca})) \begin{bmatrix} \Gamma^T \\ 0_{n\times m} \end{bmatrix} F^T(i) \begin{bmatrix} I_{n\times n} & -I_{n\times n} \end{bmatrix}
\end{aligned} \tag{4.40}
$$

$$
\begin{aligned}
& \mathbf{E}\{D(i)\}^T \\
= \ & (\beta(d_k^{sc})) \begin{bmatrix} 0_{n\times m} \\ \sigma^T \end{bmatrix} L^T(i) \begin{bmatrix} 0_{n\times n} & -I_{n\times n} \end{bmatrix}
\end{aligned} \tag{4.41}
$$

Substituting (4.37-4.40) into (4.36) and letting $X(i,r) = P^{-1}(i,r)$, we can obtain the matrix inequalities (4.34) and (4.35) in Theorem 2, which give us the desired result.

If the LMIs (4.34)(4.35) are feasible then the gain matrices F_i and L_i do exist.

Remark 4.3.1 *It is important to note that there are no products of unknown matrix $P(i,r)$ with controller parameter K_i and observer parameter L_i. Therefore, the condition (4.35) is easy to check using the existing efficient interior-point method.*

4.3.2 Stability Analysis with Partially Known Transition Probability Matrices

In Markov systems, the transition probabilities of the jumping process are crucial and all of them must be completely known a priori. However, the likelihood to obtain the complete knowledge on the transition probabilities is questionable and the cost is probably high. Thus, it is significant and challenging to further study more general jump systems with partially known transition probabilities from control perspectives, especially with time-varying delays included. More recently, some attention have been already drawn to the class of systems with time delays for both continuous-time and discrete-time. In [60] authors focused on the issue related to the stability of network control systems with packets loss. Packet-losses were modeled using two different processes. In [81]-[62], authors described the stability analysis and stabilization problems for a class of discrete-time Markov jump linear systems with partially known transition probabilities and time-varying delays were investigated. In [78], authors, studied in detail the problem concerning the H_∞ estimation for a class of Markov jump linear systems (MJLS) with varying transition probabilities (TPs) in discrete-time domain. Two types of variations were considered in the finite set of time-varying TPs: arbitrary variation and stochastic variation. However, this type analysis is so far not being fully implemented in the case of networked predictive control system.

Consider the partially unknown transition probabilities matrices, ρ and λ as

$$\rho = \begin{bmatrix} \rho_{11} & \rho_{12} & ? \\ ? & \rho_{22} & \rho_{32} \\ \rho_{31} & ? & ? \end{bmatrix}$$

$$\lambda = \begin{bmatrix} ? & \lambda_{12} & ? \\ ? & ? & \lambda_{23} \\ ? & ? & ? \end{bmatrix}$$

Where the unknown elements are represented by "?". For notation clarity, $\forall i, r \in \mathcal{I} = \{1, 2, \cdots, \mathbb{N}\}$, we denote

$\mathcal{I}_{\mathcal{K}}^{i} \triangleq \{j \text{ :if } \rho_{ij} \text{ is known}\}$, $\mathcal{I}_{u\mathcal{K}}^{i} \triangleq \{j \text{ :if } \rho_{ij} \text{ is unknown }\}$

$\mathcal{I}_{\mathcal{K}}^{r} \triangleq \{q\text{:if } \lambda_{rq} \text{ is known }\}$, $\mathcal{I}_{u\mathcal{K}}^{r} \triangleq \{n \text{ :if } \lambda_{rq} \text{ is unknown}\}$

Moreover if $\mathcal{I}_{\mathcal{K}}^{i} \neq 0$, it is further described as

$\mathcal{I}_{\mathcal{K}}^{i} = \{\mathcal{K}_1^i, \cdots, \mathcal{K}_m^i\}, \quad 1 \leq m \leq N$

Similarly,

$\mathcal{I}_{\mathcal{K}}^{r} = \{\mathcal{K}_1^r, \cdots, \mathcal{K}_m^r\}, \quad 1 \leq m \leq N$

where $\mathcal{K}_m^i$ and $\mathcal{K}_m^r$ represents the m^{th} known element with the index $\mathcal{K}_m^i$ and $\mathcal{K}_m^r$ in the i^{th} row of matrix ρ and r^{th} row of matrix λ, respectively. Also, we denote $\rho_{\mathcal{K}}^{i} \triangleq \sum_{j\in\mathcal{I}_{\mathcal{K}}^{i}} \rho_{ij}$, $\lambda_{\mathcal{K}}^{r} \triangleq \sum_{q\in\mathcal{I}_{\mathcal{K}}^{r}} \lambda_{rq}$, $P_{\mathcal{K}}^{(i,r)} \triangleq \sum_{j\in\mathcal{I}_{\mathcal{K}}^{i}} \sum_{q\in\mathcal{I}_{\mathcal{K}}^{r}} \lambda_{rq}\rho_{ij}P(j,q)$ and $P_{\mathcal{UK}}^{(i,r)} \triangleq \sum_{j\in\mathcal{I}_{\mathcal{UK}}^{i}} \sum_{q\in\mathcal{I}_{\mathcal{UK}}^{r}} \lambda_{rq}\rho_{ij}P(j,q)$

***Theorem 3.3*:** Consider the system (4.14) with partially known transition probability matrices. Then there exists a controller (4.5) such that the resulting closed-loop system is exponentially mean square stable if there exist matrices $P(i,r)^T = P(i,r) > 0, Q^T = Q > 0, \Psi_{(i,r)}, R_u, S_u, M_u, u = 1,2,3$ and $\Phi_{1u(i,r))}$ for $u = 1,2,3,\cdots,7$, Such that inequalities (4.43) and (4.44) holds.

Proof: In order to simplify the analysis, we represent $\mathbf{E}\{(A(i)+B(i,r)+C(i,r)+D(i))\}$ as Ω and $\bar{P}(i,r) = \bar{P}, A(i) = A, \quad B(i,r) = B, \quad C(i,r) = C$ and $D(i) = D$.

Therefore we have

$$
\begin{aligned}
\Psi_{11(i,r)} &= \Omega^T \bar{P} \Omega \\
\Psi_{12(i,r)} &= \Omega^T \bar{P} B \\
\Psi_{13(i,r)} &= \Omega^T \bar{P} C \\
\Psi_{14(i,r)} &= \Omega^T \bar{P} D
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{E}^{-}_{(11)} &= -R_1 + M_1^T - \Omega^T \bar{P} B \\
\mathbf{E}^{-}_{(12)} &= -R_2 + M_2^T - \Omega^T \bar{P} C \\
\mathbf{E}^{-}_{(13)} &= -R_3 + M_3^T - \Omega^T \bar{P} D \\
\mathbf{E}^{-}_{(14)} &= -M_1 - M_1^T + \mathbf{E}\{(B^T \bar{P} B\} \\
\mathbf{E}^{-}_{(15)} &= -M_2 - M_2^T + \mathbf{E}\{(C^T \bar{P} C\} \\
\mathbf{E}^{-}_{(16)} &= -M_3 - M_3^T + \mathbf{E}\{(D^T \bar{P} D\}
\end{aligned}
$$

Therefore equation (4.22) can be written as

$$
\mathbf{E}_{(i,r)} \triangleq \begin{bmatrix} \varpi_1 & \varpi_2 \end{bmatrix} < 0 \tag{4.42}
$$

Where

$$
\varpi_1 = \begin{bmatrix}
\Omega^T \bar{P} \Omega + \Psi_{i,r} & -R_1 + S_1^T & -R_2 + S_2^T & -R_3 + S_3^T \\
\bullet & -S_1 - S_1^T - Q & 0 & -S_1 - M_1^T \\
\bullet & \bullet & -S_2 - S_2^T - Q & 0 \\
\bullet & \bullet & \bullet & -S_3 - S_3^T - Q \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet
\end{bmatrix}
$$

$$\varpi_2 = \begin{bmatrix} \mathbf{E}^{-}_{(11)} & \mathbf{E}^{-}_{(12)} & \mathbf{E}^{-}_{(13)} \\ 0 & 0 & 0 \\ -S_2 - M_2^T & 0 & 0 \\ 0 & -S_3 - M_3^T & 0 \\ \mathbf{E}^{-}_{(14)} & \mathbf{E}\{(B^T \bar{P} C\} & 0 \\ \bullet & \mathbf{E}^{-}_{(15)} & \mathbf{E}(C^T \bar{P} D) \\ \bullet & \bullet & \mathbf{E}^{-}_{(16)} \end{bmatrix}$$

We know that $\bar{P} = P_{\mathcal{K}}^{(i,r)} \cup P_{\mathcal{UK}}^{(i,r)}$

where $P_{\mathcal{K}}^{(i,r)} \triangleq \sum_{j\in\mathcal{I}_{\mathcal{K}}^i} \sum_{q\in\mathcal{I}_{\mathcal{K}}^r} \lambda_{rq}\rho_{ij}P(j,q)$ and $P_{\mathcal{UK}}^{(i,r)} \triangleq \sum_{j\in\mathcal{I}_{\mathcal{UK}}^i} \sum_{q\in\mathcal{I}_{\mathcal{UK}}^r} \lambda_{rq}\rho_{ij}P(j,q)$

Equation (4.42) can be rewritten as

$$\mathbf{E}_{(i,r)} \triangleq \sum_{j\in\mathcal{I}_{\mathcal{K}}^i} \sum_{q\in\mathcal{I}_{\mathcal{K}}^r} \lambda_{rq}\rho_{ij} \begin{bmatrix} \varpi_1 & \varpi_2 \end{bmatrix} + \sum_{j\in\mathcal{I}_{\mathcal{UK}}^i} \sum_{q\in\mathcal{I}_{\mathcal{UK}}^r} \lambda_{rq}\rho_{ij} \begin{bmatrix} \varpi_3 & \varpi_4 \end{bmatrix}$$

Where

$$\varpi_1 = \begin{bmatrix} \Omega^T P(j,q)\Omega + \Psi_{i,r} & -R_1 + S_1^T & -R_2 + S_2^T & -R_3 + S_3^T \\ \bullet & -S_1 - S_1^T - Q & 0 & -S_1 - M_1^T \\ \bullet & \bullet & -S_2 - S_2^T - Q & 0 \\ \bullet & \bullet & \bullet & -S_3 - S_3^T - Q \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{bmatrix}$$

$$\varpi_2 = \begin{bmatrix} \mathbf{E}_{(11)} & \mathbf{E}_{(12)} & \mathbf{E}_{(13)} \\ 0 & 0 & 0 \\ -S_2 - M_2^T & 0 & 0 \\ 0 & -S_3 - M_3^T & 0 \\ \mathbf{E}_{(14)} & \mathbf{E}\{(B^T P(j,q) & 0 \\ \bullet & \mathbf{E}_{(15)} & \mathbf{E}(C^T P(j,q)D) \\ \bullet & \bullet & \mathbf{E}_{(16)} \end{bmatrix}$$

$$\varpi_3 = \begin{bmatrix} \Omega^T P(j,q)\Omega + \Psi_{i,r} & -R_1 + S_1^T & -R_2 + S_2^T & -R_3 + S_3^T \\ \bullet & -S_1 - S_1^T - Q & 0 & -S_1 - M_1^T \\ \bullet & \bullet & -S_2 - S_2^T - Q & 0 \\ \bullet & \bullet & \bullet & -S_3 - S_3^T - Q \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{bmatrix}$$

$$\varpi_4 = \begin{bmatrix} \mathbf{E}_{(11)} & \mathbf{E}_{(12)} & \mathbf{E}_{(13)} \\ 0 & 0 & 0 \\ -S_2 - M_2^T & 0 & 0 \\ 0 & -S_3 - M_3^T & 0 \\ \mathbf{E}_{(14)} & \mathbf{E}\{(B^T P(j,q)C\} & 0 \\ \bullet & \mathbf{E}_{(15)} & \mathbf{E}(C^T P(j,q)D) \\ \bullet & \bullet & \mathbf{E}_{(16)} \end{bmatrix}$$

Where

$$
\begin{aligned}
\mathbf{E}_{(11)} &= -R_1 + M_1^T - \Omega^T \bar{P}(j,q)B \\
\mathbf{E}_{(12)} &= -R_2 + M_2^T - \Omega^T \bar{P}(j,q)C \\
\mathbf{E}_{(13)} &= -R_3 + M_3^T - \Omega^T \bar{P}(j,q)D \\
\mathbf{E}_{(14)} &= -M_1 - M_1^T + \mathbf{E}\{(B^T \bar{P}(j,q)B\} \\
\mathbf{E}_{(15)} &= -M_2 - M_2^T + \mathbf{E}\{(C^T \bar{P}(j,q)C\} \\
\mathbf{E}_{(16)} &= -M_3 - M_3^T + \mathbf{E}\{(D^T \bar{P}(j,q)D\}
\end{aligned}
$$

$$
\mathbf{E}_{(i,r)} \triangleq \begin{bmatrix} \varpi_1 & \varpi_2 \end{bmatrix} + \sum_{j \in \mathcal{I}^i_{\mathcal{UK}}} \sum_{q \in \mathcal{I}^r_{\mathcal{UK}}} \lambda_{rq}\rho_{ij} \begin{bmatrix} \varpi_3 & \varpi_4 \end{bmatrix}
$$

Where

$$
\varpi_1 = \begin{bmatrix}
\Omega^T P_{\mathcal{K}}^{(i,r)} \Omega + \Psi_{i,r} & -R_1 + S_1^T & -R_2 + S_2^T & -R_3 + S_3^T \\
\bullet & -S_1 - S_1^T - Q & 0 & -S_1 - M_1^T \\
\bullet & \bullet & -S_2 - S_2^T - Q & 0 \\
\bullet & \bullet & \bullet & -S_3 - S_3^T - Q \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet
\end{bmatrix}
$$

$$\varpi_2 = \begin{bmatrix} \mathbf{E}_{(11)\mathcal{K}} & \mathbf{E}_{(12)\mathcal{K}} & \mathbf{E}_{(13)\mathcal{K}} \\ 0 & 0 & 0 \\ -S_2 - M_2^T & 0 & 0 \\ 0 & -S_3 - M_3^T & 0 \\ \mathbf{E}_{(14)\mathcal{K}} & \mathbf{E}\{(B^T P_{\mathcal{K}}^{(i,r)} C\} & 0 \\ \bullet & \mathbf{E}_{(15)\mathcal{K}} & \mathbf{E}(C^T P_{\mathcal{K}}^{(i,r)} D) \\ \bullet & \bullet & \mathbf{E}_{(16)\mathcal{K}} \end{bmatrix}$$

$$\varpi_3 = \begin{bmatrix} \Omega^T P(j,q)\Omega + \Psi_{i,r} & -R_1 + S_1^T & -R_2 + S_2^T & -R_3 + S_3^T \\ \bullet & -S_1 - S_1^T - Q & 0 & -S_1 - M_1^T \\ \bullet & \bullet & -S_2 - S_2^T - Q & 0 \\ \bullet & \bullet & \bullet & -S_3 - S_3^T - Q \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{bmatrix}$$

$$\varpi_4 = \begin{bmatrix} \mathbf{E}_{(11)} & \mathbf{E}_{(12)} & \mathbf{E}_{(13)} \\ 0 & 0 & 0 \\ -S_2 - M_2^T & 0 & 0 \\ 0 & -S_3 - M_3^T & 0 \\ \mathbf{E}_{(14)} & \mathbf{E}\{(B^T P(j,q)C\} & 0 \\ \bullet & \mathbf{E}_{(15)} & \mathbf{E}(C^T P(j,q)D) \\ \bullet & \bullet & \mathbf{E}_{(16)} \end{bmatrix}$$

Where

$$\begin{aligned} \mathbf{E}_{(11)\mathcal{K}} &= -R_1 + M_1^T - \Omega^T P_{\mathcal{K}}^{(i,r)} B \\ \mathbf{E}_{(12)\mathcal{K}} &= -R_2 + M_2^T - \Omega^T P_{\mathcal{K}}^{(i,r)} C \\ \mathbf{E}_{(13)\mathcal{K}} &= -R_3 + M_3^T - \Omega^T P_{\mathcal{K}}^{(i,r)} D \\ \mathbf{E}_{(14)\mathcal{K}} &= -M_1 - M_1^T + \mathbf{E}\{(B^T P_{\mathcal{K}}^{(i,r)} B\} \\ \mathbf{E}_{(15)\mathcal{K}} &= -M_2 - M_2^T + \mathbf{E}\{(C^T P_{\mathcal{K}}^{(i,r)} C\} \\ \mathbf{E}_{(16)\mathcal{K}} &= -M_3 - M_3^T + \mathbf{E}\{(D^T P_{\mathcal{K}}^{(i,r)} D\} \end{aligned}$$

Therefore if one have

$$\begin{bmatrix} \varpi_1 & \varpi_2 \end{bmatrix} < 0 \tag{4.43}$$

$$\begin{bmatrix} \varpi_3 & \varpi_4 \end{bmatrix} < 0 \tag{4.44}$$

Where

$$\varpi_1 = \begin{bmatrix} \Omega^T P_{\mathcal{K}}^{(i,r)} \Omega + \Psi_{i,r} & -R_1 + S_1^T & -R_2 + S_2^T & -R_3 + S_3^T \\ \bullet & -S_1 - S_1^T - Q & 0 & -S_1 - M_1^T \\ \bullet & \bullet & -S_2 - S_2^T - Q & 0 \\ \bullet & \bullet & \bullet & -S_3 - S_3^T - Q \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{bmatrix}$$

$$\varpi_2 = \begin{bmatrix} \mathbf{E}_{(11)\mathcal{K}} & \mathbf{E}_{(12)\mathcal{K}} & \mathbf{E}_{(13)\mathcal{K}} \\ 0 & 0 & 0 \\ -S_2 - M_2^T & 0 & 0 \\ 0 & -S_3 - M_3^T & 0 \\ \mathbf{E}_{(14)\mathcal{K}} & \mathbf{E}\{(B^T P_{\mathcal{K}}^{(i,r)} C\} & 0 \\ \bullet & \mathbf{E}_{(15)\mathcal{K}} & \mathbf{E}(C^T P_{\mathcal{K}}^{(i,r)} D) \\ \bullet & \bullet & \mathbf{E}_{(16)\mathcal{K}} \end{bmatrix}$$

$$\varpi_3 = \begin{bmatrix} \Omega^T P(j,q)\Omega + \Psi_{i,r} & -R_1 + S_1^T & -R_2 + S_2^T & -R_3 + S_3^T \\ \bullet & -S_1 - S_1^T - Q & 0 & -S_1 - M_1^T \\ \bullet & \bullet & -S_2 - S_2^T - Q & 0 \\ \bullet & \bullet & \bullet & -S_3 - S_3^T - Q \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{bmatrix}$$

$$\varpi_4 = \begin{bmatrix} \mathbf{E}_{(11)} & \mathbf{E}_{(12)} & \mathbf{E}_{(13)} \\ 0 & 0 & 0 \\ -S_2 - M_2^T & 0 & 0 \\ 0 & -S_3 - M_3^T & 0 \\ \mathbf{E}_{(14)} & \mathbf{E}\{(B^T P(j,q)C\} & 0 \\ \bullet & \mathbf{E}_{(15)} & \mathbf{E}(C^T P(j,q)D) \\ \bullet & \bullet & \mathbf{E}_{(16)} \end{bmatrix} < 0$$

then we have $\mathbf{E}_{(i,m)} < 0$, hence the system is exponentially mean square stable under partially known transition probabilities, which is concluded from the obvious fact that no knowledge on $\rho_{ij} \forall j \in \mathcal{I}^i_{\mathcal{UK}}$ and $\lambda_{rq} \forall n \in \mathcal{I}^q_{\mathcal{UK}}$, is required in (4.43) and (4.44). Thus, for $\rho^i_{\mathcal{K}}, \lambda^r_{\mathcal{K}} \neq 0$ and $\rho^i_{\mathcal{K}}, \lambda^r_{\mathcal{K}} = 0$,, respectively, one can readily obtain (4.42), since if $\rho^i_{\mathcal{K}}, \lambda^r_{\mathcal{K}} = 0$ the conditions (4.43), (4.44) will reduce to (4.44). This completes the proof.

The delay dependent gain matrices F_i and L_i are given as:

Consider that $\daleth_i = Q^{-1}\mathbf{E}\{D(i)\}^T$, then

$$\daleth_i = (\beta(d_k^{sc}))Q \begin{bmatrix} 0_{p\times n} \\ \sigma^T \end{bmatrix} L^T(i) \begin{bmatrix} 0_{n\times n} & -I_{n\times n} \end{bmatrix} \tag{4.45}$$

So, the observer gain L_i is obtained by premultiplying $\begin{bmatrix} 0_{n\times m} & \sigma^T \end{bmatrix} Q^{-1}$ and post multiplying $\begin{bmatrix} 0_{n\times n} & -I_{n\times n} \end{bmatrix}^T$ to both side of (4.45).

where

$$\daleth_i = \begin{cases} (\beta(d_k^{sc}))Q \begin{bmatrix} 0_{p\times n} \\ \sigma^T \end{bmatrix} L^T(i) \begin{bmatrix} 0_{n\times n} & -I_{n\times n} \end{bmatrix} & \text{if} \quad d_k^{sc} = i > 0 \\ 0 & \text{if} \quad d_k^{sc} = i = 0 \end{cases}$$

For the controller F_i, consider $\chi_{i,r} = Q^{-1}\mathbf{E}\{B(i,r)\}^T$, then

$$\chi_{i,r} = (1 - \alpha(d_k^{ca})) \begin{bmatrix} \Gamma^T \\ 0_{n\times m} \end{bmatrix} F^T(i) \begin{bmatrix} I_{n\times n} & -I_{n\times m} \end{bmatrix} \tag{4.46}$$

The controller F_i is obtained by premultiplying $\begin{bmatrix} \Gamma^T & 0_{n\times m} \end{bmatrix} Q^{-1}$ and post multiplying $\begin{bmatrix} I_{n\times n} & -I_{n\times n} \end{bmatrix}^T$ to both side of (4.46).

where

$$\chi_{i,0} = \begin{cases} (1-\alpha(d_k^{ca}))Q \begin{bmatrix} \Gamma^T \\ 0_{n\times m} \end{bmatrix} F^T(i) \begin{bmatrix} I_{n\times n} & -I_{n\times m} \end{bmatrix} & \text{if} \quad d_k^{ca} = r = 0 \\ 0 & \text{if} \quad d_k^{ca} = r > 0 \end{cases}$$

4.4 Numerical Example

To illustrate the effectiveness of the proposed methodology, We apply the results obtained above to milling machine tool described by the system model[69]

$$\phi = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -18.18 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -17.86 \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} 0 & 0 \\ 515.38 & 0 \\ 0 & 0 \\ 0 & 517.07 \end{bmatrix}, \quad \sigma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

By selecting the sampling time=.1sec, we obtain the discretized system as,

$$\phi = \begin{bmatrix} 1 & .0461 & 0 & 0 \\ 0 & .1624 & 0 & 0 \\ 0 & 0 & 1 & .0466 \\ 0 & 0 & 0 & .1676 \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} 1.5287 & 0 \\ 23.7 & 0 \\ 0 & 1.5458 \\ 0 & 24.0982 \end{bmatrix}, \quad \sigma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The eigenvalues of the above mentioned milling machine tool system are 1.0, .16124, 1.0, and 0.1676. Hence, the discrete time system is unstable. Figure 4.2 shows the open loop response of the above system without any control signal applied. The simulation was carried out on Matlab R2010, wherein, LMIs were solved to obtain the gain matrices F_i and L_i.

Transition probability matrices, ρ_{ij} and λ_{mn} used to model the delay in measurement and actuation channels respectively are,

$$\rho_{ij} = \begin{bmatrix} .32 & .28 & .4 \\ .6 & .4 & 0 \\ .099 & .01 & .89 \end{bmatrix}$$

$$\lambda_{rq} = \begin{bmatrix} .2 & .8 \\ .6 & .2 \end{bmatrix}$$

Then by using (4.45) and (4.46), we obtain the controller gain F_i and observer gain L_i as

$$F_0 = \begin{bmatrix} -0.00776 & -0.0114 & 0.005 & -0.0003 \\ -.000808 & .000609 & .00423 & .00914765 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} -0.00043 & -0.00001 & -0.0087 & -0.0112 \\ -.000131 & .000972 & .00062 & .000374 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} -0.00087 & -0.00033 & -0.0008 & -0.0098 \\ -.000934 & .000531 & .00941 & .00021 \end{bmatrix}$$

$$L_0 = \begin{bmatrix} -0.0006 & 0.00384 & 0.0007 & -0.000111 \\ 0.00009 & -0.0001 & 0.0005 & 0.00009 \end{bmatrix}^T$$

$$L_1 = \begin{bmatrix} 0.000654 & 0.00384 & 0.000657 & -0.000101 \\ -0.00032 & -0.000311 & -0.0008 & 0.00009 \end{bmatrix}^T$$

$$L_2 = \begin{bmatrix} -0.00076 & 0.00004 & 0.00693 & -0.0077 \\ -0.00065 & 0.00003 & -0.00053 & -0.0001 \end{bmatrix}^T$$

Figures 4.3 and 4.4 depicts the state responses of the milling machine tool,

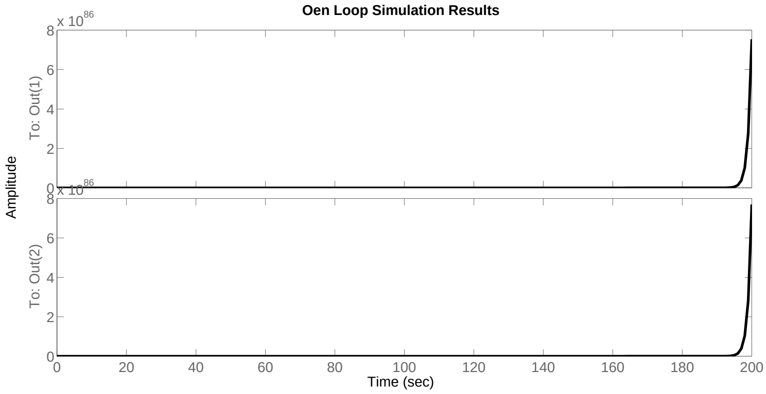

Figure 4.2: Open loop response of the system

when the transition probability matrices are completely known

Now consider the case of partially known transition matrices.

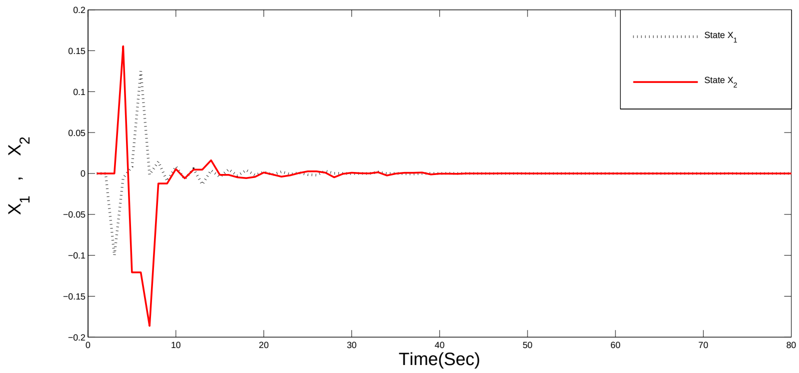

Figure 4.3: State response X_1 and X_2 of system

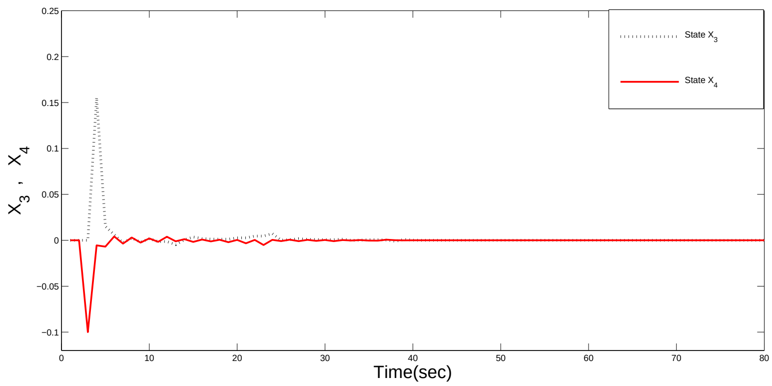

Figure 4.4: State response X_3 and X_4 of system

Let

$$\rho_{ij} = \begin{bmatrix} ? & .28 & ? \\ ? & ? & 0 \\ .099 & .01 & .89 \end{bmatrix}$$

$$\lambda_{rq} = \begin{bmatrix} ? & .8 \\ ? & ? \end{bmatrix}$$

The controller gain F_i and observer gain L_i obtained in this case are,

$$F_0 = \begin{bmatrix} -0.00032 & 0.0004 & 0.0001 & -0.0003 \\ -.000853 & .000065 & .004005 & .00910655 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} -0.00043 & -0.00001 & -0.0087 & -0.0112 \\ -.000131 & .000972 & .00062 & .000374 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} -0.00087 & 0.00033 & -0.0008 & 0.0098 \\ .000934 & .000531 & .00941 & .00021 \end{bmatrix}$$

$$L_0 = \begin{bmatrix} -0.0006 & 0.00397 & 0.000987 & 0.000221 \\ 0.00009 & 0.0001 & 0.0087 & 0.0065009 \end{bmatrix}^T$$

$$L_1 = \begin{bmatrix} 0.000654 & 0.00384 & 0.000657 & -0.00051 \\ -0.00032 & -0.00651 & -0.000641 & 0.00841 \end{bmatrix}^T$$

$$L_2 = \begin{bmatrix} -0.00076 & 0.00004 & 0.00693 & -0.0087 \\ -0.00760 & -0.00521 & -0.06532 & -0.0087 \end{bmatrix}^T$$

The state responses of the system in the case of partially known transition matrices are as shown in fig (4.5-4.6)

From the simulation graphs shown in fig.(4.3-4.6), it can be concluded that, the novel control algorithm developed in this paper, takes less time for the NCS to converge to zero and stabilize the system.

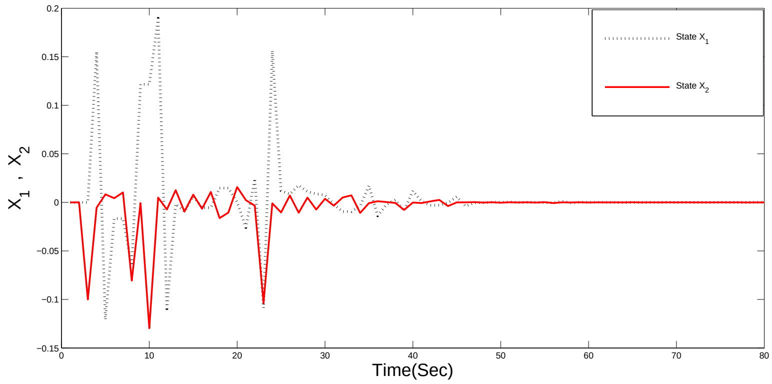

Figure 4.5: State response X_1 and X_2 of system with partially known transition matrices

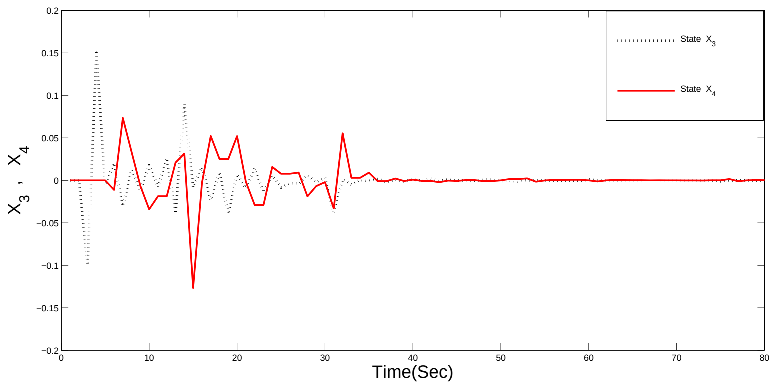

Figure 4.6: State response X_3 and X_4 of system with partially known transition matrices

4.5 Conclusion

This paper provides a novel dynamic output feedback based networked predictive control scheme for Markov jump linear systems subjected to network phenomena such as packet dropout on S/C and C/A side. In this paper we have considered an important features of NCSs to transmit a packet of data set at the same time. Based on this feature the networked predictive scheme was proposed. The stability analysis and controller design was given based on designed technique. The two important contributions of this paper are 1. to consider packet dropouts both in the network connecting plant and observer and also in the network connecting observer and controller. 2. controller gain(F) and observer gain(L) are decided based on the number of packet dropped out on the sensor controller side (d_k^{sc}). A numerical example has been given to show the effectiveness of developed method. It is worth mentioning that the results proposed in this paper can be extended for the case with input/output constrains, which deserve further investigation.

Chapter 5

CONCLUSIONS AND FUTURE WORK

This thesis has presented, a new control design strategies for networked control systems to deal with the control problem that arises when control loops are closed over a communication network. The stability and performance of the physical system gets affected by packets dropouts and network induced time delays. Markov chain model used in the analysis has the captivating property of making models of possible network delays. These Markov chain models are more practical and realistic in nature when compared to the other delay models.

The recent reported research activities in networked control were concerned with three models of the network delays, namely,

1. constant delays,
2. Random delay, which is independent from transfer to transfer,
3. Random delay, with probability distributions governed by an underlying Markov chain.

Constant delays is one of the simplest way to model the network delay is to model it as being constant for all transfers in the communication network. This type of model gives the satisfactory response inmost of the cases. Much research has been reported in this area. Several investigators have addressed the problems of constant delay compensation in closed loop control systems.

Various activities in the networked control system like, transmission of the waiting messages, retransmission, networks collisions, etc. are usually not synchronized with each other, as a result, the delays occur in random fashion. This type of networked delays are modeled are as independent delays. These type of delays are independent from the time delays. However, in a real communication system these transfer time are usually correlated with the last transfer delay. These delays constitutes a Markov model with only one state. Many researchers has discussed in detail the effects of these random delays on the system stability and proposed future improvements

The most advanced one that is used to generates the probability distributions of the time delays is the Markov chain model. Effects such as varying network load can also be modeled by making the Markov chain do a transition every time a transfer is done in the communication network. Much research has been directed towards the study of effect of communication packet losses modeled as Markov chain in the feedback loop of a control system. Initially, the stability conditions and controller designs were derived based on the assumption that packet dropout exists only in the sensor-to-controller (S/C) side. The effect of controller-to-actuator (C/A) packet dropouts is neglected due to the complicated NCSs modeling. Recently, some results were obtained, were NCSs with packet dropouts on both S/C and C/A sides are considered.

In Markov systems, the transition probabilities of the jumping process are crucial and all of them must be completely known a priori. However, the likelihood to obtain the complete knowledge on the transition probabilities is questionable and the cost is probably high. A recent research trend has shown that a fair amount of research work has been focused on the issue related to the stability of network control systems with packets loss, where transition probability matrices are partially known. However to the best of the authors knowledge, the analysis with unknown transition probability matrices describing the packets dropouts on both the side S/C and C/A respectively has so far not been completely addressed in the literature of NCSs.

The work of Jing Wu and Tongwen Chen [59] has been extended in this thesis, where in an optimistic assumption of known transition probability matrices was considered. The work in [59] was developed based on the assumption that all the elements of the transition matrices ρ_{ij} and λ_{mn} are completely known in advance. Prior knowledge of these matrices was assumed to be compulsory.

In this thesis a class of system is considered with partially known transition probability matrices. stability analysis and controller design is carried out for such type of systems. The simulation results shows that the developed technique is very well able to stabilize the NCSs where the transition probability matrices are partially known. It was also concluded that more the information we have about the transition probability matrices, better was the performance of the system. This shows that their exist a tradeoff between cost of obtaining the transition probabilities and system performance.

In the next part of the thesis we considered the designing of networked predictive control technique with dynamic output feed back. In the past few decades, model predictive control (MPC), has received much attention from the researchers in dealing with the problems of NCSs like, transmission losses or packet losses. Extensive application of MPC controller is reported in the control of many industrial processes like, pulp and paper processing ,distillation and oil fractionation, and so on. Even though a wide range of research has recently been reported in the area of networked predictive control systems, many researchers ignored some of important feature of networked control like, the ability of communication channels to transfer a packet of data set at the same time, which cannot be accomplished in traditional control system. A full opportunity of this NCSs feature was taken and a new novel dynamic output feedback based networked predictive control technique was proposed in chapter **4** to deal with network control systems with data packets dropouts in both measurement channel (sensor -controller S/C) and actuation channel (controller actuator- C/A). This was basically an extension of the work done in [70] by Jing Wu et al. In [70], networked model predictive controller was designed based on the assumption that the states are directly available from the plant. In practicality however, the states are not readily available from the plant to be used by the controller. They are to be estimated. So, the investigation carried out in [70] was without the consideration of observer or the state estimator. So, the work done by Jing Wu et al. [70] was enhanced by developing an improved observer based stabilizing control algorithm to estimate the states. Another important contribution was the addition of buffer in the actuation channel to store the data received from the plant which is more realistic and practical in nature. Moreover, the controller gain(F) and observer gain(L) were made to be decided based on the number of packet dropped out on the sensor controller sided_k^{sc}. Such type of considerations further improve the analysis in terms of system performance versus the packet loss.

The area of networked control is still a raw arena for research advances, and constitutes a new paradigm for control engineers to explore. Hence our work in this thesis has the great potential to be further expanded in several directions. Some of the problems not treated in this thesis are:

- With reference to Chapter 3, we considered that the elements of transition probability matrices are unknown but constant with time. However, NCSs with partially unknown transition probability matrices with time varying elements can be explored. This problem is interesting and very hard.

- The results proposed in chapter 4 can be extended for the case with input/output constrains

- It would also be interesting to investigate the stability analysis of NCSs using the theory from adaptive control.

Finally we would like to reiterate that the analysis presented in this thesis is general in the sense that no specific underlying network type, structure or operating protocol has been taken into consideration. Hence the results can be applied to wired or wireless type of networks, with the knowledge that several specific models will have to be devised for the latter.

Nomenclature

$x(k)$ - plant state vector ($\in \Re^m$)

$u(k)$ - plant control input vector ($\in \Re^n$)

$y(k)$ - plant output vector ($\in \Re^p$)

$\hat{x}(k)$ - estimated plant state vector ($\in \Re^m$)

$\hat{y}(k)$ - estimated plant output vector ($\in \Re^m$)

$e(k)$ - error vector ($\in \Re^m$)

d_k^{sc} -the quantity of packets dropped at time k on the S/C side

d_k^{ca} - the quantity of packets dropped at time k on the C/A side

T_s - sampling time

$\mathbf{E}$ - Expectation operator

ρ - Markovian transition matrix on the S/C side

λ - Markovian transition matrix on the C/A side

$\alpha(d_k^{ca})$ - control signal is successfully transmitted or not in C/A channel

$\beta(d_k^{sc})$ - output signal is successfully transmitted or not in S/C channel

P - Lyapunov positive-symmetric matrix

Q - Lyapunov positive-semi-symmetric matrix

R - Lyapunov positive-semi-symmetric matrix

F - state-feedback gain matrix ($\in \Re^{m \times n}$)

L - observer gain matrix ($\in \Re^{m \times p}$)

k - sampling instant

J - cost function

p - prediction horizon length

q - control horizon length

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Vitae

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